# Modeling Relational Event Dynamics with statnet

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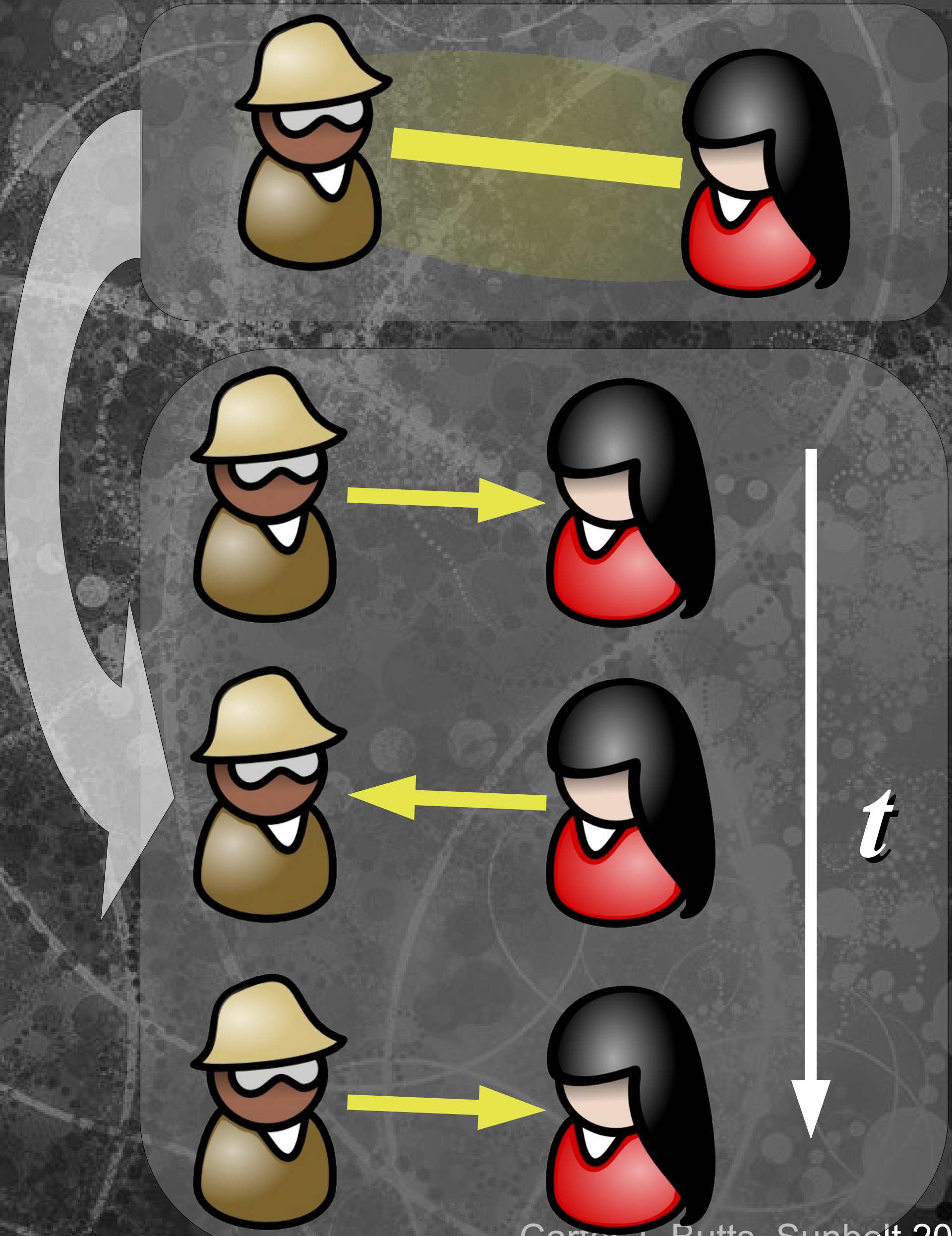
# Overview

- Content in a nutshell
  - Introduction to the use of relational event models (Butts, 2006; 2008) for the modeling interaction dynamics
  - Why this approach?
    - Fairly general
    - Principled basis for inference (estimation, model comparison, etc.) from actually existing data
    - Utilizes well-understood formalisms (event history analysis, multinomial logit)
- This workshop:
  - Introduction to modeling approach
  - Fitting dyadic relational event models
  - Model assessment and simulation

# Unpacking Networks: From Relationships to Action

- Conventional network paradigm: focus on temporally extensive relationships
- Powerful approach, but not always ideal
- Sometimes, we are interested in the social action that lies beneath the relationships....



# Actions and Relational Events

- Action: discrete event in which one entity emits a behavior directed at one or more entities in its environment
  - Useful "atomic unit" of human activity
  - Represent formally by relational events
- Relational event: $a=(i,j,k,t)$
  - $i \in \mathcal{S}$: "Sender" of event $a$; $s(a)=i$
  - $j \in \mathcal{R}$: "Receiver" of event $a$; $r(a)=j$
  - $k \in \mathcal{C}$: "Action type" ("category") for event $a$; $c(a)=k$
  - $t \in \mathbb{R}$: "Time" of event $a$; $\tau(a)=t$

# Events in Context

- Multiple actions form an event history, $A_t = \{a_i : \tau(a_i) \leq t\}$
  - Take $a_0$: $\tau(a_0)=0$ as "null action", $\tau(a_i) \geq 0$
  - Possible actions at $t$ given by $\mathrm{A}(A_t) \subseteq \mathcal{S} \times \mathcal{R} \times \mathcal{C}$
    - Forms support for next action
    - Assume here that $\mathrm{A}(A_t)$ finite, constant between actions; may be fixed, but need not be
- Goal: model $A_t$
  - Treat actions as events in continuous time
  - Hazards depend upon past history, covariates

Possible Events

Event Hazards

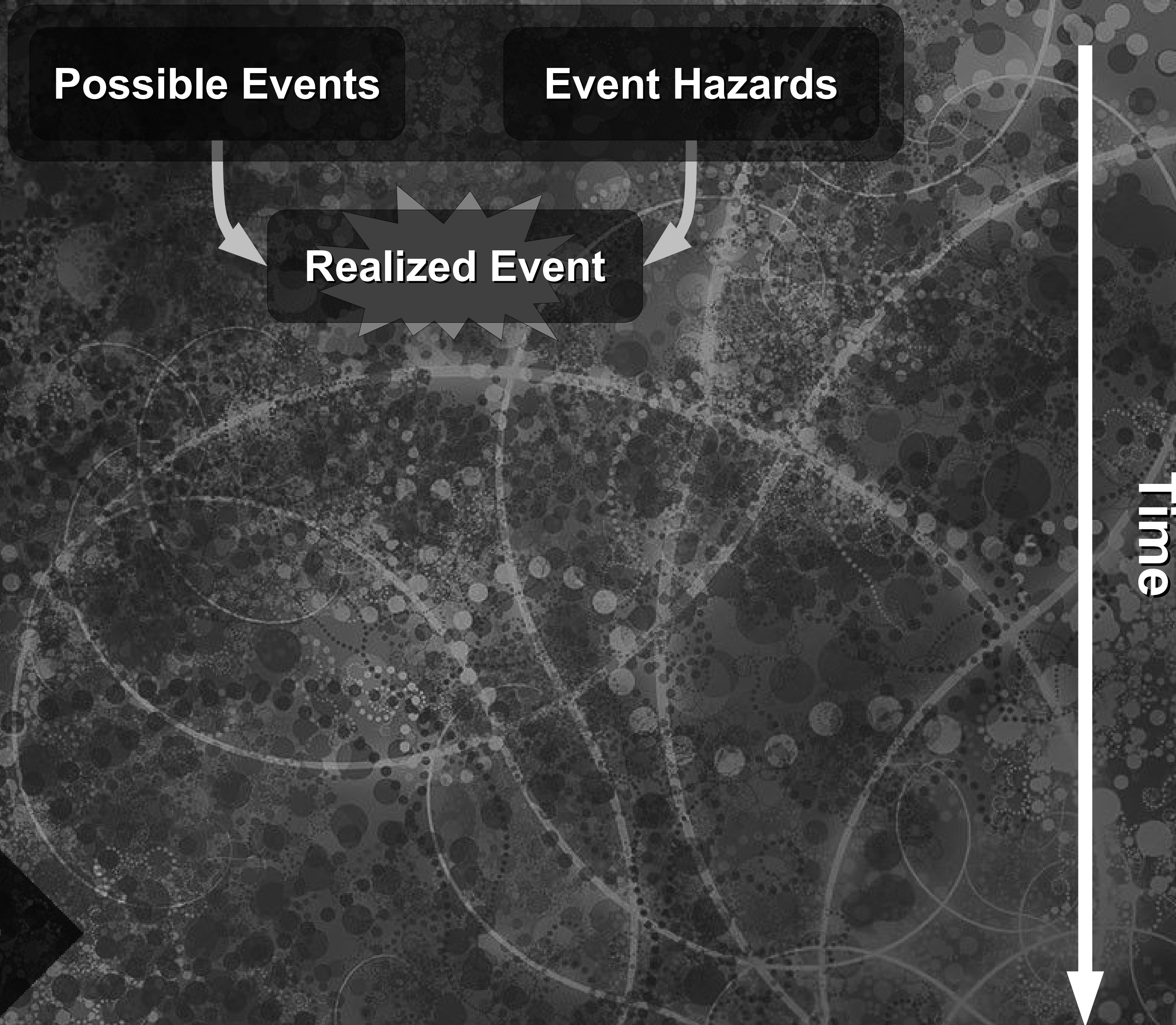
Time

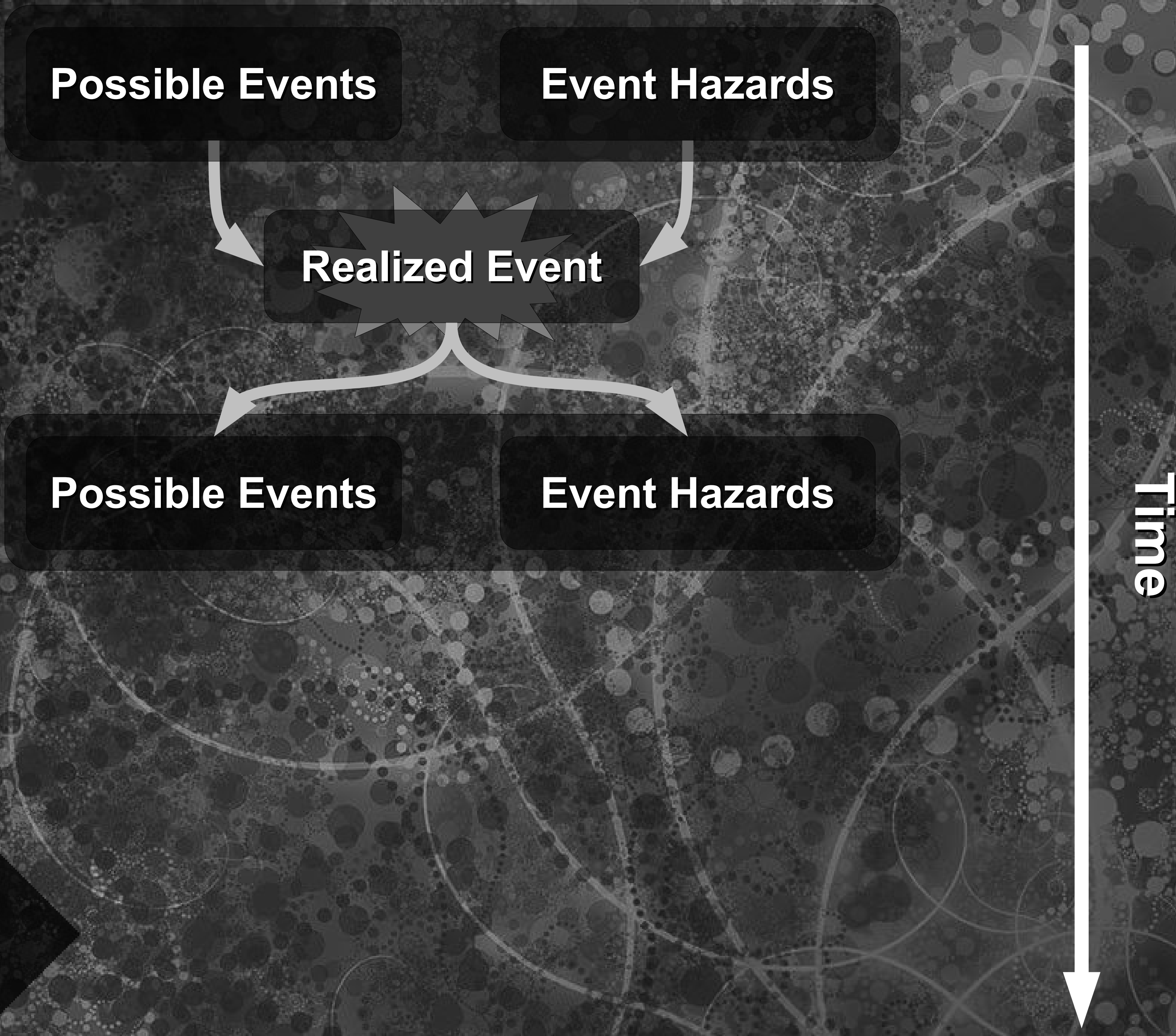
Possible Events

Event Hazards

Context

Time


Possible Events
Event Hazards
Realized Event
Time



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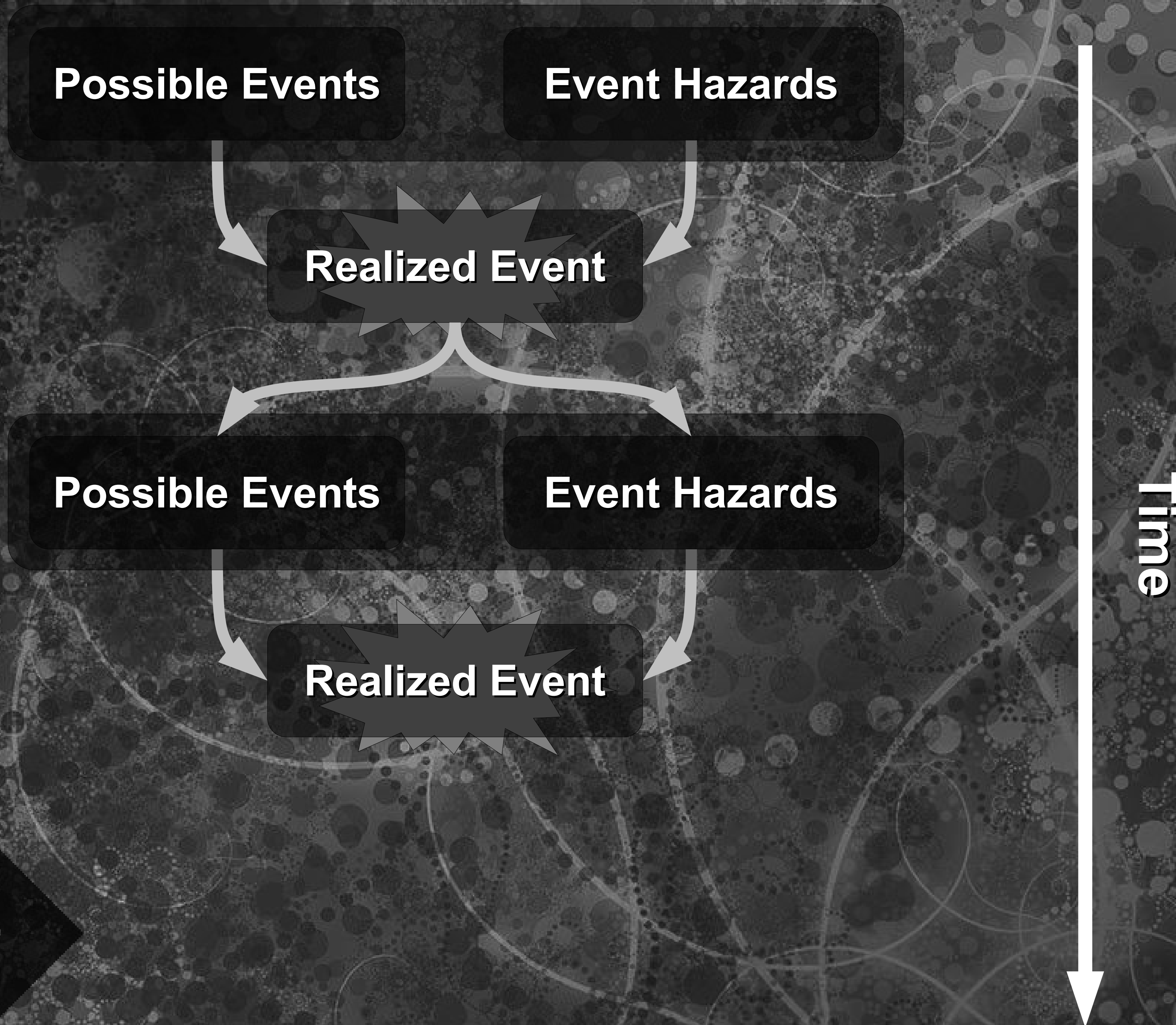
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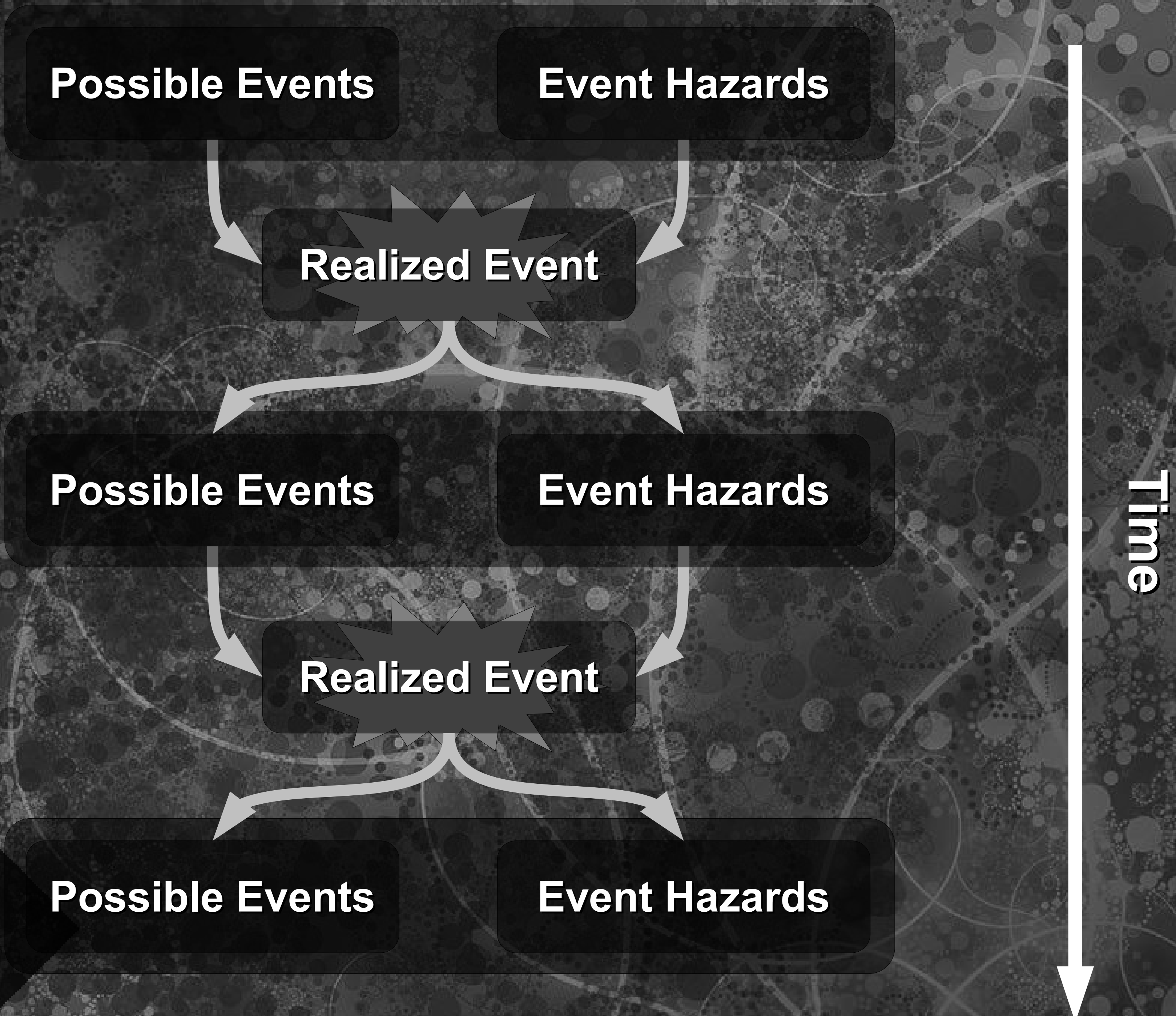

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# Event Model Likelihood: Piecewise Exponential Case

- **Natural simplifying assumption: actions arise as Poisson-like events with piecewise constant rates**
  - Intuition: hazard of each possible event is *locally* constant, given complete event history up to that point
    - Waiting times conditionally exponentially distributed
    - Rates *can* change when events transpire, but not otherwise
      - Compare to related assumption in Cox prop. hazards model
    - Possible events likewise change only when something happens
- **Can use to derive event likelihood**
  - Let $M=|A_t|$, $\tau_i=\tau(a_i)$, w/hazard function $\lambda_{a_i A_k \theta}=\lambda(a_i, A_k, \theta)$; then

$$p(A_t|\theta)=\left[\prod_{i=1}^{M}\left(\lambda_{a_i A_{\tau_{i-1}}\theta}\prod_{a'\in \mathrm{A}(A_{\tau_i})}\exp\left(-\lambda_{a' A_{\tau_{i-1}}\theta}\left[\tau_i-\tau_{i-1}\right]\right)\right)\right]\left[\prod_{a'\in \mathrm{A}(A_t)}\exp\left(-\lambda_{a' A_t \theta}\left[t-\tau_M\right]\right)\right]$$

# The Problem of Uncertain Event Timing

- Likelihood of an event sequence depends on the detailed history
  - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known
  - What if we only have (temporally) ordinal data?
- Stochastic process theory to the rescue!
  - Thm: Let $X_1,\ldots,X_n$ be independent exponential r.v. w/rate parameters $\lambda_1,\ldots,\lambda_n$. Then the probability that $x_i=\min\{x_1,\ldots,x_n\}$ is $\lambda_i/(\lambda_1+\ldots+\lambda_n)$.
  - Implication: likelihood of ordinal data is a product of multinomial likelihoods
    - Identifies rate function up to a constant factor

# Event Model Likelihood: Ordinal Timing Case

- **Using the above, we may write the likelihood of an event sequence $A_t$ as follows:**

$$p\left(A_t|\theta\right)=\prod_{i=1}^{M}\left[\frac{\lambda_{a_i A_{\tau_{i-1}}\theta}}{\sum_{a'\in \mathrm{A}\left(A_{\tau_i}\right)}\lambda_{a_i A_{\tau_{i-1}}\theta}}\right]$$

- **Dynamics governed by rate function, λ**

$$\lambda_{a A_t \theta}=\begin{cases}\exp\left(\lambda_0+\theta^T u\left(s(a),r(a),c(a),A_t,X_a\right)\right) & a\in \mathrm{A}\left(A_t\right)\\ 0 & a\notin \mathrm{A}\left(A_t\right)\end{cases}$$

  - **Where $\lambda_0$ is an arbitrary constant, $\theta\in\mathbb{R}^p$ is a parameter vector, and $u$: $(i,j,A_t,X)\rightarrow\mathbb{R}^p$ is a vector of statistics**

# Interpreting the Parameters

- In general, each unit change in $u_i$ multiplies the hazard of an associated event by $\exp(\theta_i)$
  - For ordinal time case, unit difference in $u_i$ adds unit of $\theta_i$ to log odds of $a$ vs $a'$
- Connection to multinomial choice models
  - Let $\mathrm{A}_i(A_t)$ be the set of possible actions for sender $i$ at time $t$. Then, conditional on no other event occurring before $i$ acts, the probability that $i$'s next action is $a$ is given by

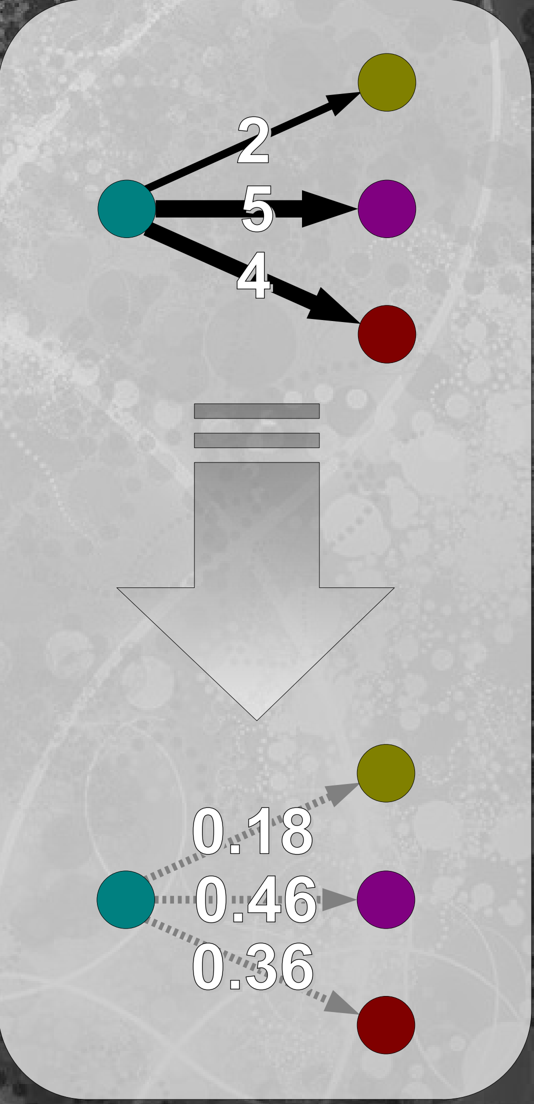
$$p(a|\theta) = \frac{\exp\left[\theta^T u\left(i, r(a), c(a), A_t, X_a\right)\right]}{\sum_{a' \in \mathrm{A}_i(A_t)} \exp\left[\theta^T u\left(i, r(a'), c(a'), A_t, X_{a'}\right)\right]}$$

# Fitting Relational Event Models

- Given $A_t$ and $u$, how do we estimate $\theta$?
  - Parameters interpretable as logged rate multipliers (in $u$)
- We have $p(A_t|\theta)$, so can conduct likelihood-based inference
  - Find MLE $\theta^*=\arg\max_\theta p(A_t|\theta)$, e.g., using a variant Newton-Rapheson or other method
  - Can also proceed in a Bayesian manner
    - Posit $p(\theta)$, work with $p(\theta|A_t)\propto p(A_t|\theta)p(\theta)$
  - Some computational challenges when |A| is large; tricks like MC quadrature needed to deal with sum of rates across support

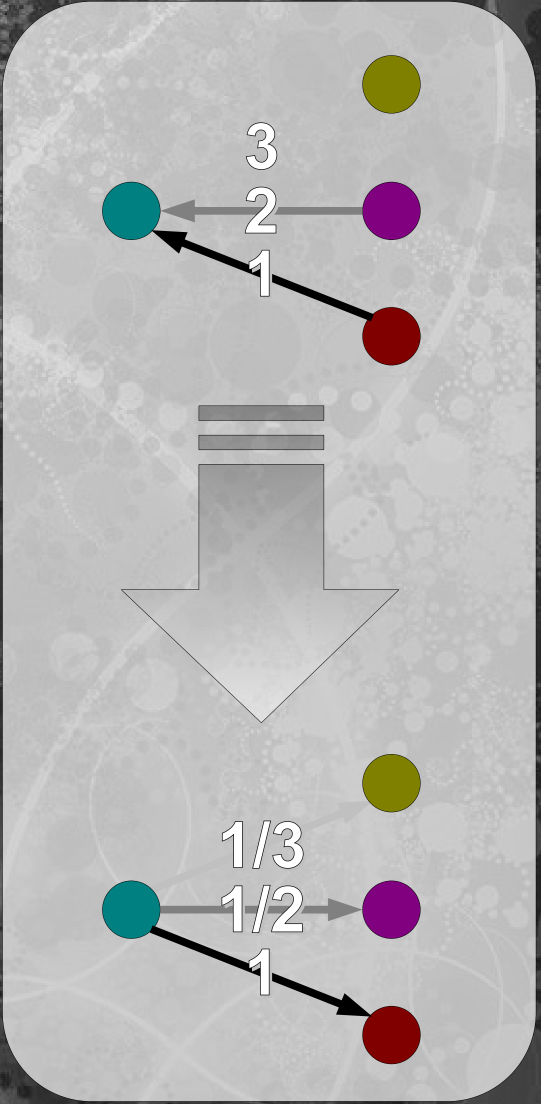
# Persistence Effects

- Inertia-like effect: past contacts may tend to become future contacts
  - Unobserved relational heterogeneity
  - Availability to memory
  - (Compare w/autocorrelation terms in an AR process)
- Simple implementation: fraction of previous contacts as predictor
  - Log-rate of $(i,j)$ contact adjusted by $\theta d_{ij}/d_i$



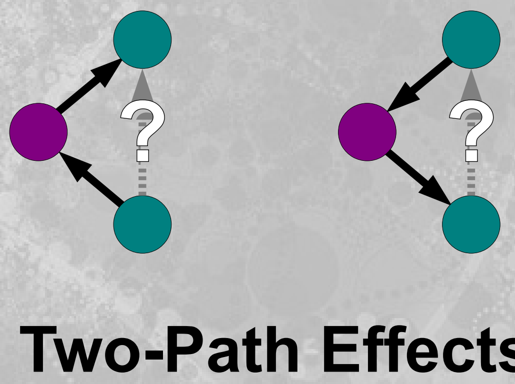
# Recency/Ordering Effects

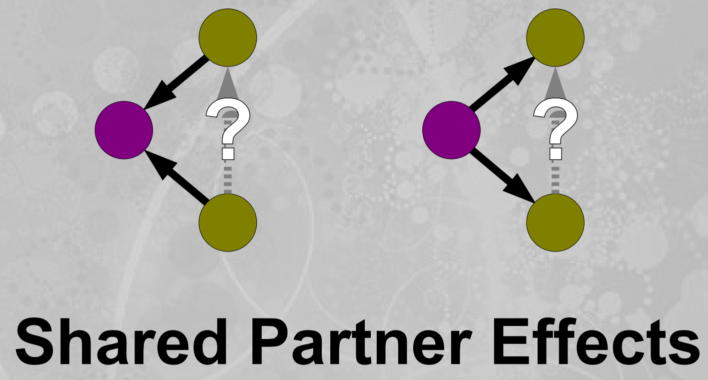
- Ordering of past contact potentially affects future contact
  - Reciprocity norms
  - Recency effects (salience)
- Simple parameterization: dyadic contact ordering effect
  - Previous incoming contacts ranked
    - Non-contacts treated as rank ∞
  - Log-rate of outgoing $(i,j)$ contact adjusted by $\theta(1/\mathrm{rank}_{ji})$



# Triadic/Clustering Effects

- Can also control for endogenous triadic mechanisms
  - Two-path effects
    - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
    - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)
  - Shared partner effects
    - Past outbound shared partners lead to/inhibit direct contact (common reference)
    - Past inbound shared partners lead to/inhibit direct contact (common contact)

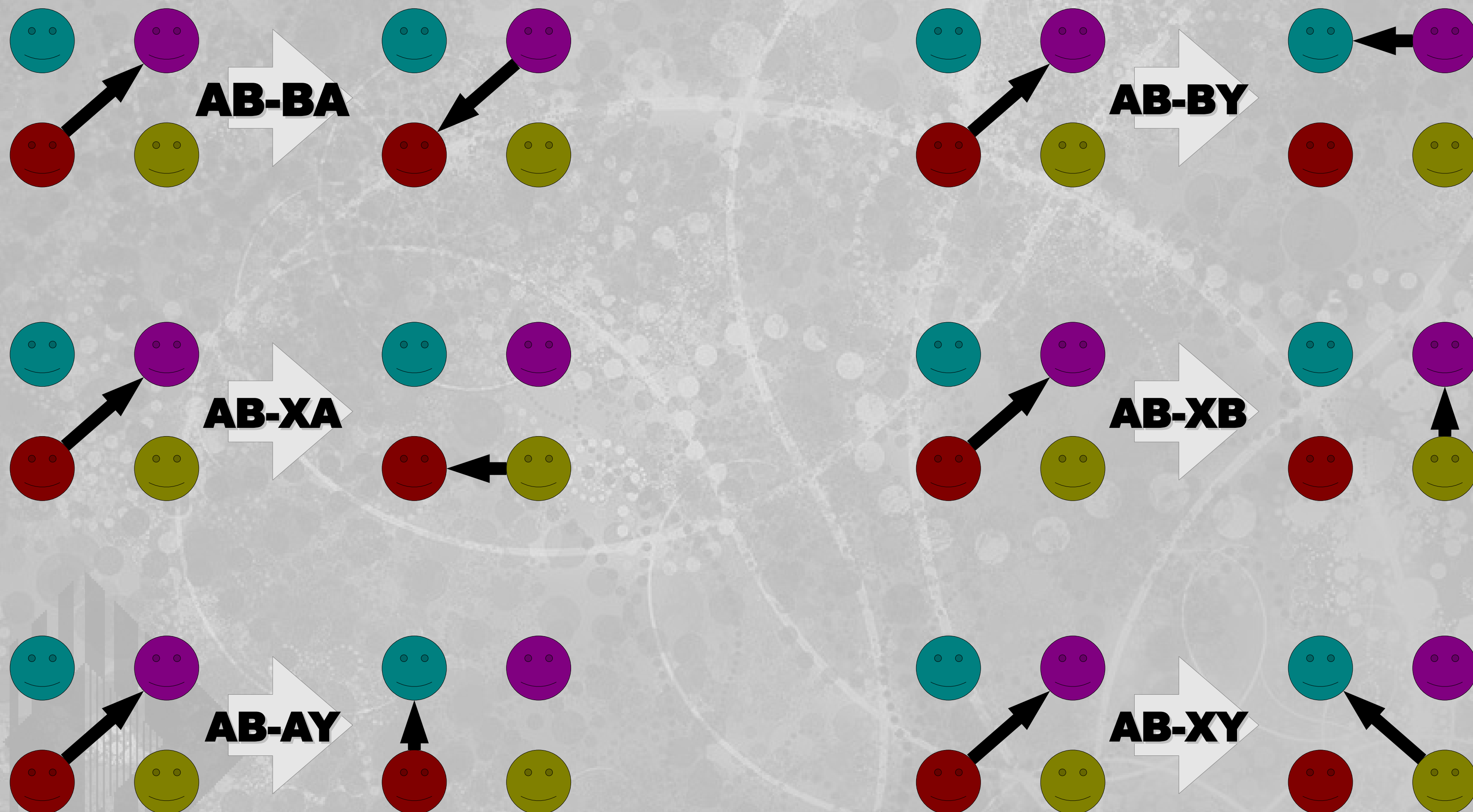


# Participation Shifts

- Proposal of Gibson (2003) for studying conversational dynamics
  - Classify actors into *senders*, *receivers*, and *bystanders*
  - When roles change, a *participation shift* ("P-shift") is said to occur
  - Study conversational dynamics by examining the incidence of P-shifts
- P-shift typology
  - For dyadic communication, 6 possible P-shifts; allowing indefinite targets expands set to 13
  - Can compute observed, potential shifts given an event sequence

# Dyadic P-Shifts, Illustrated



# Preferential Attachment

- Past interactive activity affects tendency to receive action
  - E.g., emergent coordination roles
    - Exposure-based saliency ("who's out there?")
    - Practice/specialization (efficiency)
- Implement via past total degree effect on hazard of receipt
  - Fraction of all past calls due to *i* as effect for all *j* to *i* events