Modeling Relational Event Dynamics with statnet

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2023 Sunbelt Conference, Portland, OR

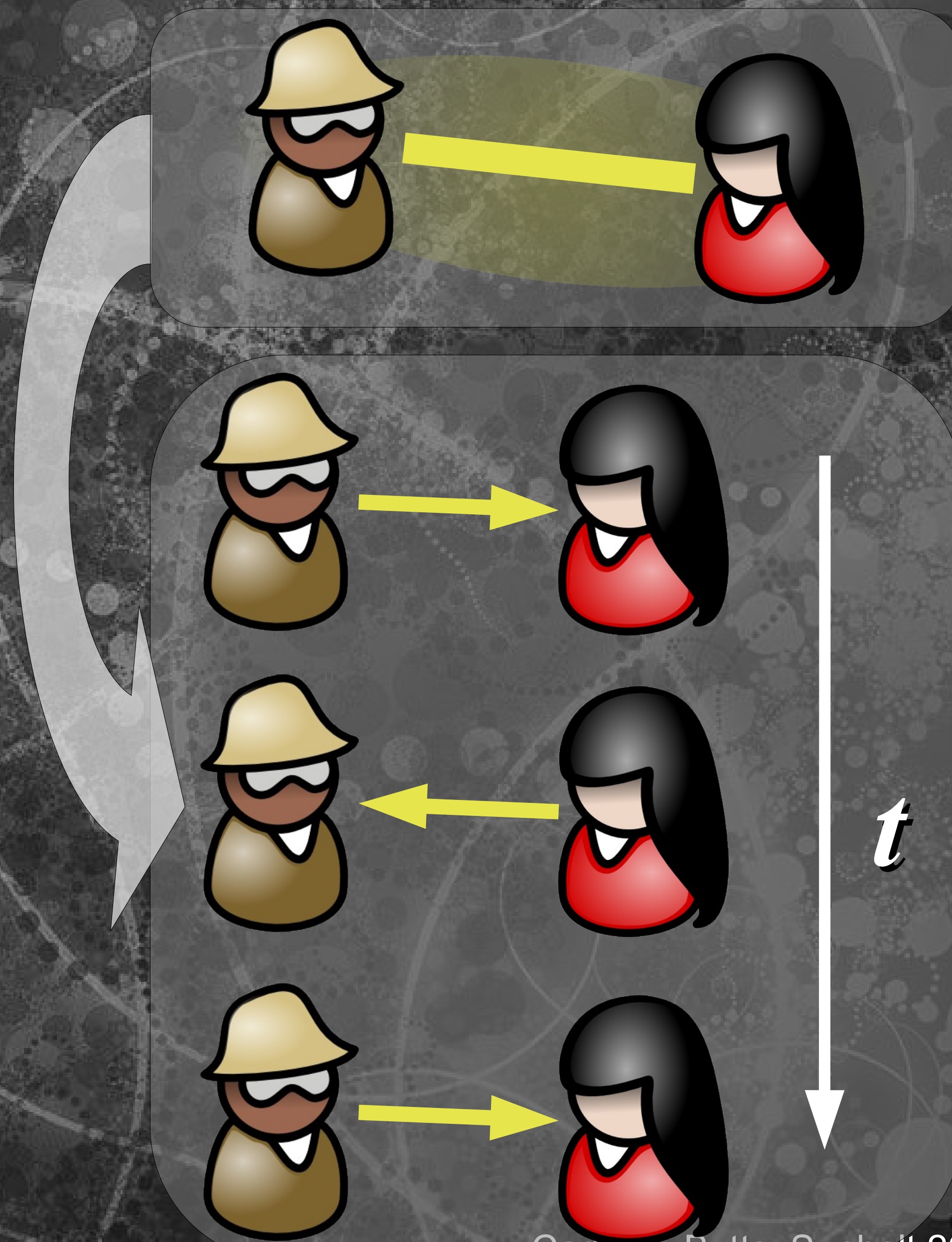
This work was supported by NSF awards IIS-0331707, CMS-064257, and SES-1826589, DOD ONR award N00014-8-1-1015, and NIH awards 1R01HD068395-01 and 1R01GM144964-01.

Overview

- Content in a nutshell
  - Introduction to the use of relational event models (Butts, 2006; 2008) for the modeling interaction dynamics
  - Why this approach?
    - Fairly general
    - Principled basis for inference (estimation, model comparison, etc.) from actually existing data
    - Utilizes well-understood formalisms (event history analysis, multinomial logit)
- This workshop:
  - Introduction to modeling approach
  - Fitting dyadic relational event models
  - Model assessment and simulation

Unpacking Networks: From Relationships to Action

- **Conventional network paradigm: focus on temporally extensive relationships**
- **Powerful approach, but not always ideal**
- **Sometimes, we are interested in the social action that lies beneath the relationships....**



Actions and Relational Events

- Action: discrete event in which one entity emits a behavior directed at one or more entities in its environment
  - Useful "atomic unit" of human activity
  - Represent formally by relational events
- Relational event: $a=(i,j,k,t)$
  - $i \in \mathcal{S}$: "Sender" of event a; $s(a)=i$
  - $j \in \mathcal{R}$: "Receiver" of event a; $r(a)=j$
  - $k \in \mathcal{C}$: "Action type" ("category") for event a; $c(a)=k$
  - $t \in \mathbb{R}$: "Time" of event a; $\tau(a)=t$

Events in Context

- **Multiple actions form an event history,** $A_t=\{a_i: \tau(a_i)\leq t\}$
  - **Take** a_0**:** $\tau(a_0)=0$ **as "null action",** $\tau(a_i)\geq 0$
  - **Possible actions at** t **given by** $\mathbf{A}(A_t)\subseteq\mathcal{S}\times\mathcal{R}\times\mathcal{C}$
    - **Forms support for next action**
    - **Assume here that** $\mathbf{A}(A_t)$ **finite, constant between actions; may be fixed, but need not be**
- **Goal: model** A_t
  - **Treat actions as events in continuous time**
  - **Hazards depend upon past history, covariates**

Possible Events

Event Hazards

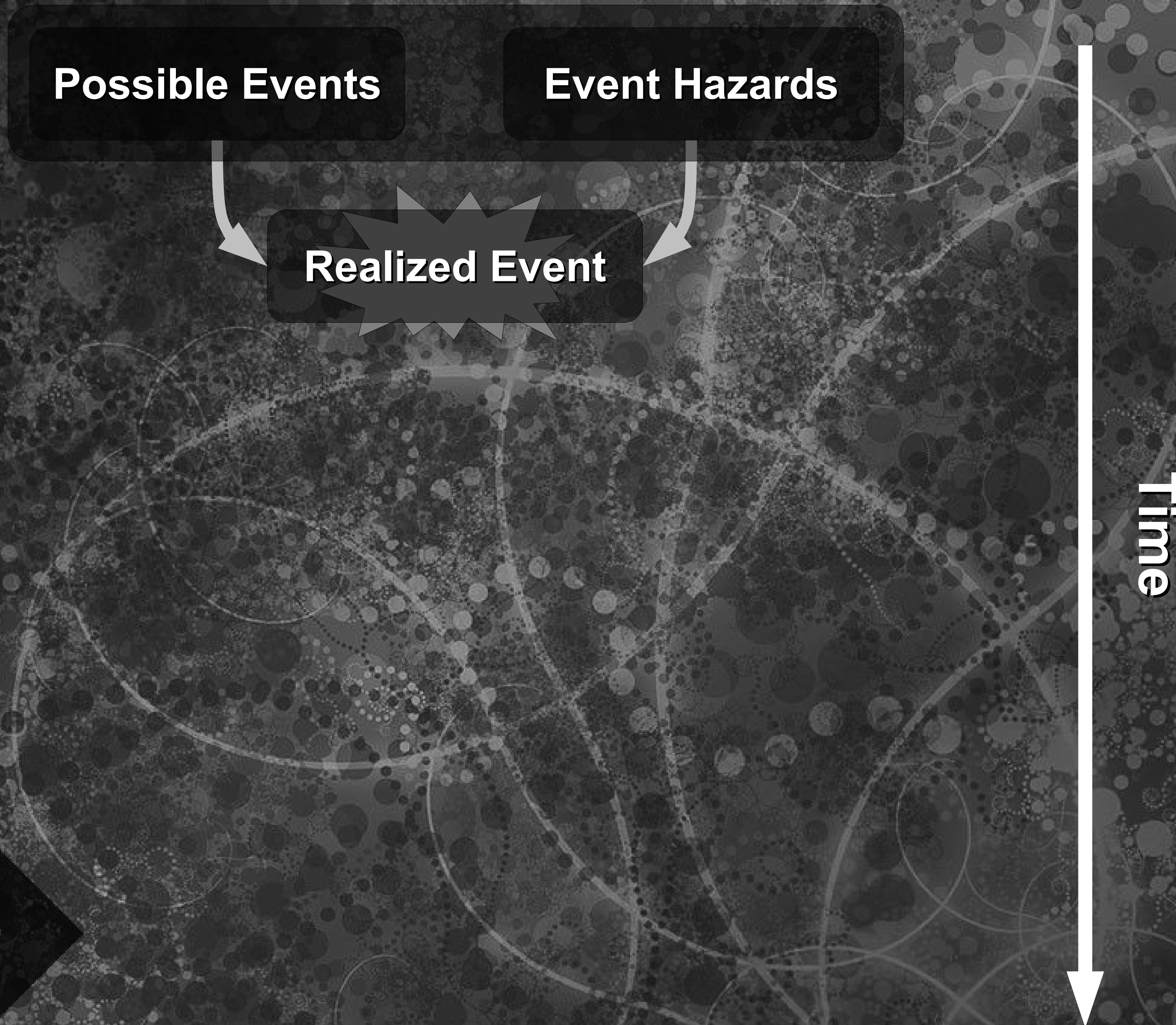
Time

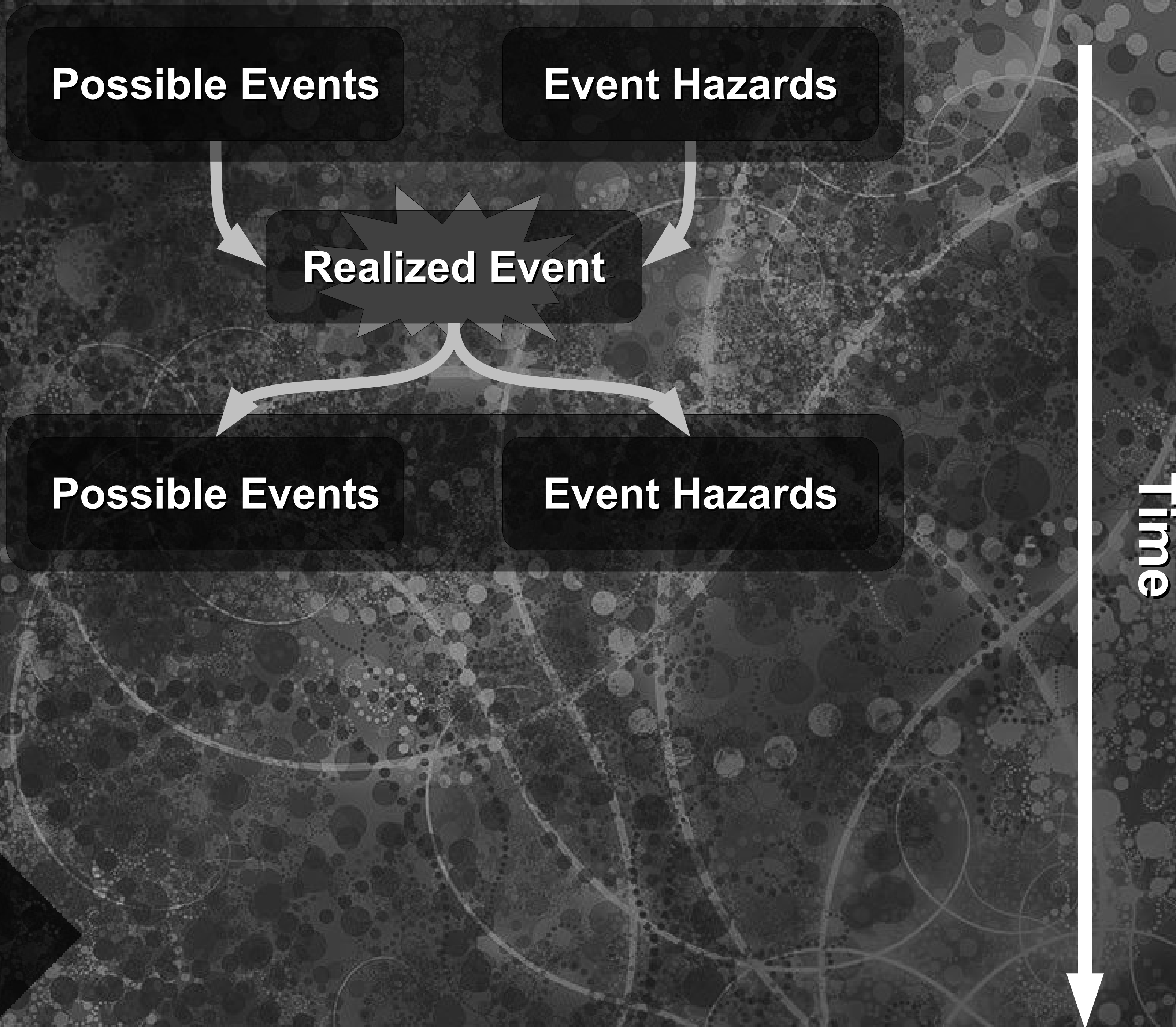
Possible Events

Event Hazards

Context

Time


Possible Events
Event Hazards
Realized Event
Time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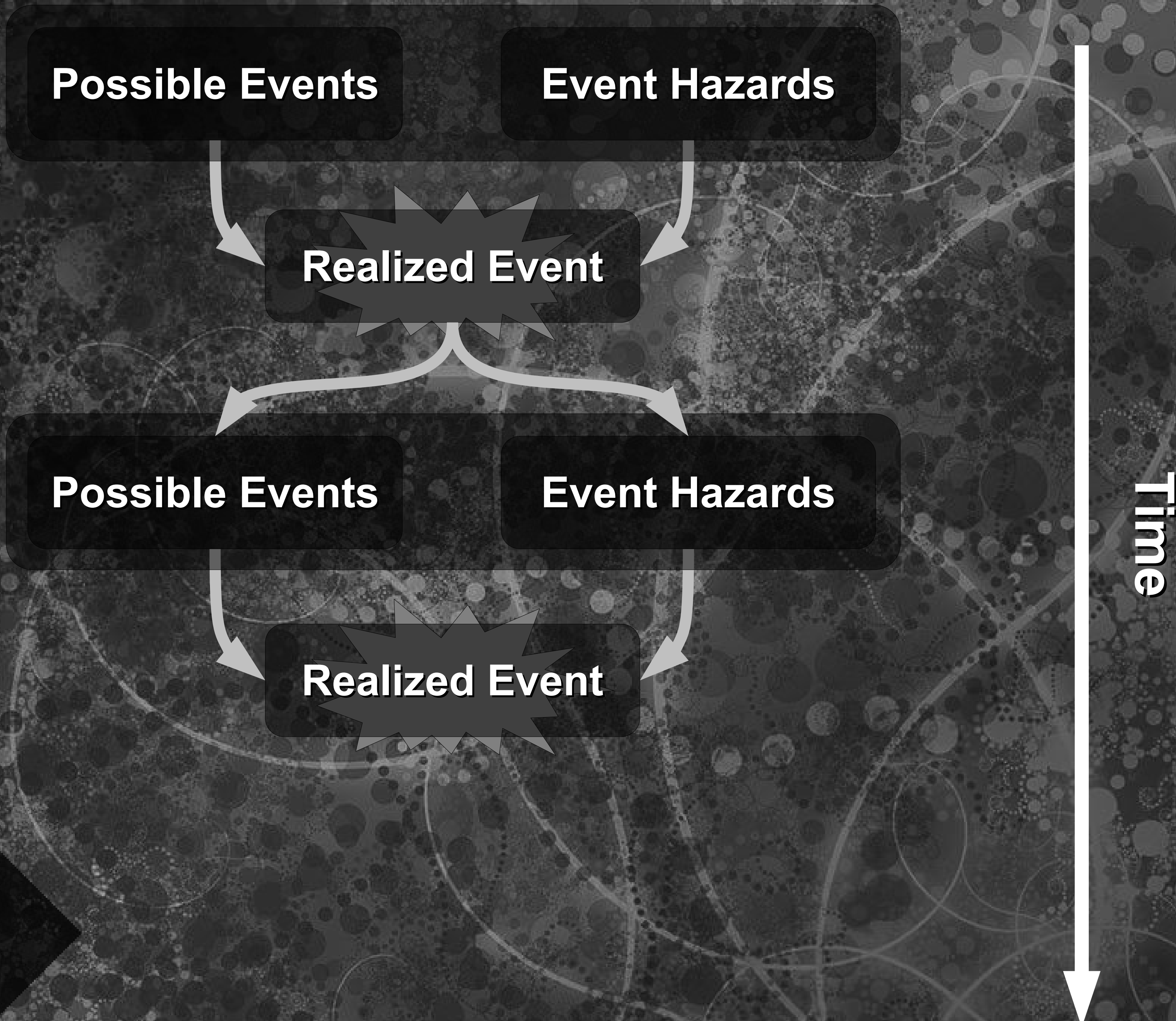
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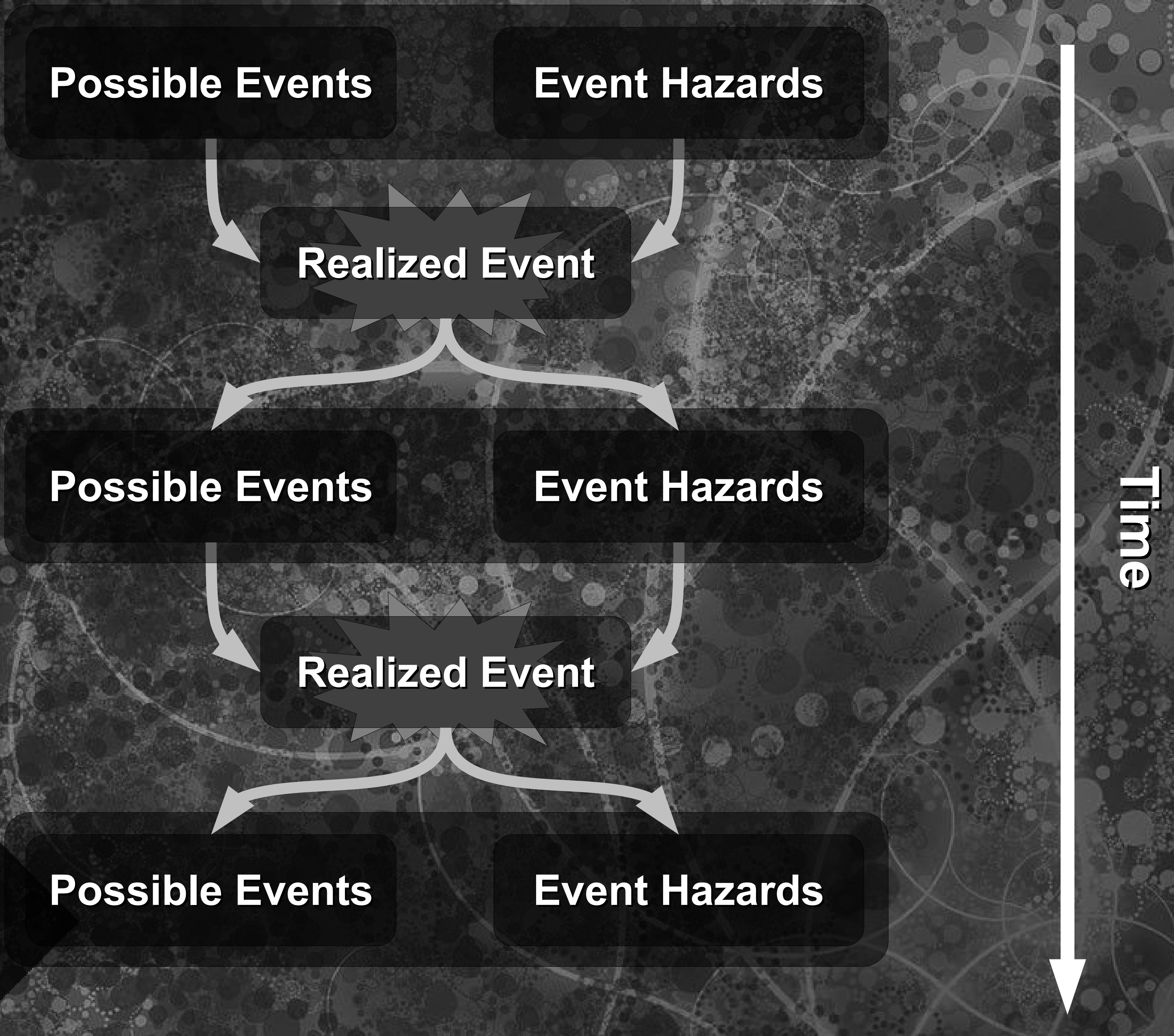

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Event Model Likelihood: Piecewise Exponential Case

- Natural simplifying assumption: actions arise as Poisson-like events with piecewise constant rates
  - Intuition: hazard of each possible event is *locally* constant, given complete event history up to that point
    - Waiting times conditionally exponentially distributed
    - Rates *can* change when events transpire, but not otherwise
      - Compare to related assumption in Cox prop. hazards model
    - Possible events likewise change only when something happens
- Can use to derive event likelihood
  - Let $M=|A_t|$, $\tau_i=\tau(a_i)$, w/hazard function $\lambda_{a_i A_k \theta}=\lambda(a_i, A_k, \theta)$; then

$$p(A_t|\theta)=\left[\prod_{i=1}^{M}\left(\lambda_{a_i A_{\tau_{i-1}}\theta}\prod_{a'\in\mathrm{A}(A_{\tau_i})}\exp\left(-\lambda_{a' A_{\tau_{i-1}}\theta}\left[\tau_i-\tau_{i-1}\right]\right)\right)\right]\left[\prod_{a'\in\mathrm{A}(A_t)}\exp\left(-\lambda_{a' A_t \theta}\left[t-\tau_M\right]\right)\right]$$

The Problem of Uncertain Event Timing

- Likelihood of an event sequence depends on the detailed history
  - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known
  - What if we only have (temporally) ordinal data?
- Stochastic process theory to the rescue!
  - Thm: Let $X_1,\ldots,X_n$ be independent exponential r.v. w/rate parameters $\lambda_1,\ldots,\lambda_n$. Then the probability that $x_i=\min\{x_1,\ldots,x_n\}$ is $\lambda_i/(\lambda_1+\ldots+\lambda_n)$.
  - Implication: likelihood of ordinal data is a product of multinomial likelihoods
    - Identifies rate function up to a constant factor

Event Model Likelihood: Ordinal Timing Case

- Using the above, we may write the likelihood of an event sequence A_t as follows:

$$p(A_t|\theta)=\prod_{i=1}^{M}\left[\frac{\lambda_{a_i A_{\tau_{i-1}}\theta}}{\sum_{a'\in \mathrm{A}(A_{\tau_i})}\lambda_{a_i A_{\tau_{i-1}}\theta}}\right]$$

- Dynamics governed by rate function, λ

$$\lambda_{a A_t \theta}=\begin{cases}\exp\left(\lambda_0+\theta^T u\left(s(a),r(a),c(a),A_t,X_a\right)\right) & a\in \mathrm{A}(A_t)\\ 0 & a\notin \mathrm{A}(A_t)\end{cases}$$

  - Where λ_0 is an arbitrary constant, $\theta \in \mathbb{R}^p$ is a parameter vector, and u: $(i,j,A_t,X)\rightarrow \mathbb{R}^p$ is a vector of statistics

Interpreting the Parameters

- In general, each unit change in u_i multiplies the hazard of an associated event by $\exp(\theta_i)$
  - For ordinal time case, unit difference in u_i adds unit of θ_i to log odds of a vs a'
- Connection to multinomial choice models
  - Let $\mathrm{A}_i(A_t)$ be the set of possible actions for sender i at time t. Then, conditional on no other event occurring before i acts, the probability that i's next action is a is given by

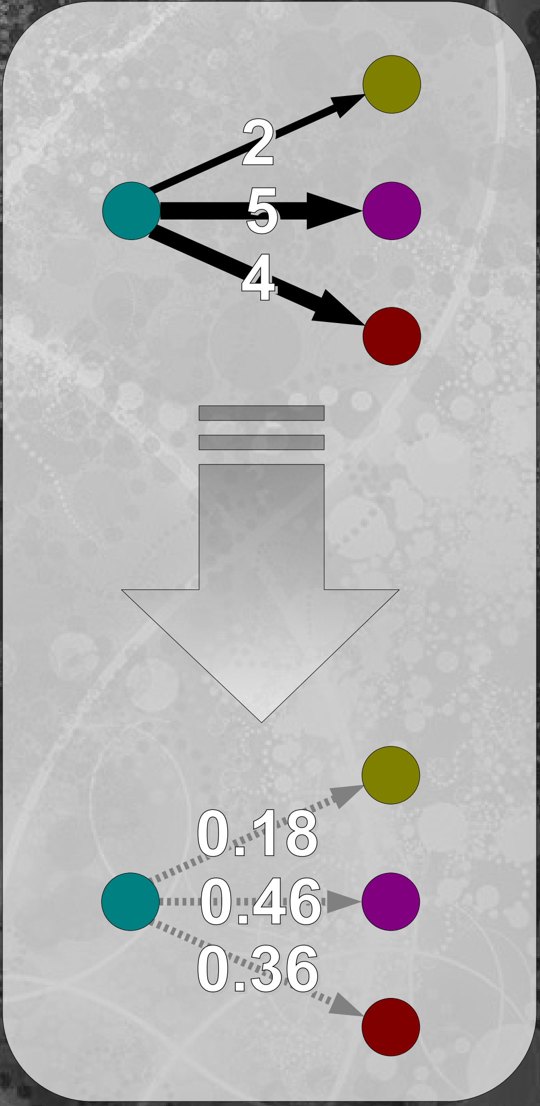
$$p(a|\theta)=\frac{\exp\left[\theta^T u\left(i,r(a),c(a),A_t,X_a\right)\right]}{\sum_{a'\in \mathrm{A}_i\left(A_t\right)}\exp\left[\theta^T u\left(i,r(a'),c(a'),A_t,X_{a'}\right)\right]}$$

Fitting Relational Event Models

- Given A_t and u, how do we estimate θ?
  - Parameters interpretable as logged rate multipliers (in u)
- We have $p(A_t|\theta)$, so can conduct likelihood-based inference
  - Find MLE $\theta^* = \arg\max_\theta p(A_t|\theta)$, e.g., using a variant Newton-Rapheson or other method
  - Can also proceed in a Bayesian manner
    - Posit $p(\theta)$, work with $p(\theta|A_t) \propto p(A_t|\theta)p(\theta)$
  - Some computational challenges when |**A**| is large; tricks like MC quadrature needed to deal with sum of rates across support

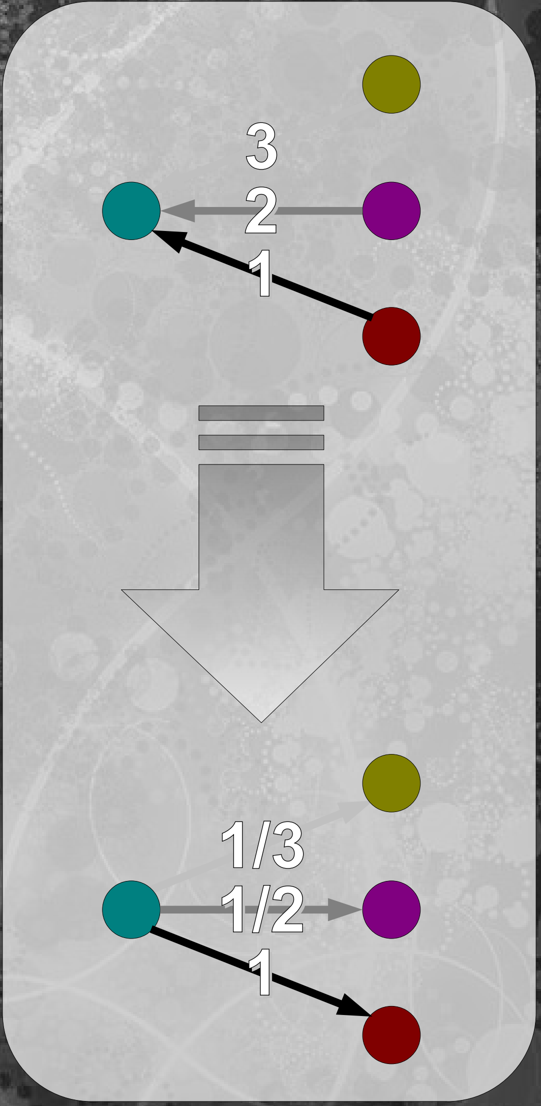
Persistence Effects

- Inertia-like effect: past contacts may tend to become future contacts
  - Unobserved relational heterogeneity
  - Availability to memory
  - (Compare w/autocorrelation terms in an AR process)
- Simple implementation: fraction of previous contacts as predictor
  - Log-rate of (i,j) contact adjusted by $\theta d_{ij}/d_i$



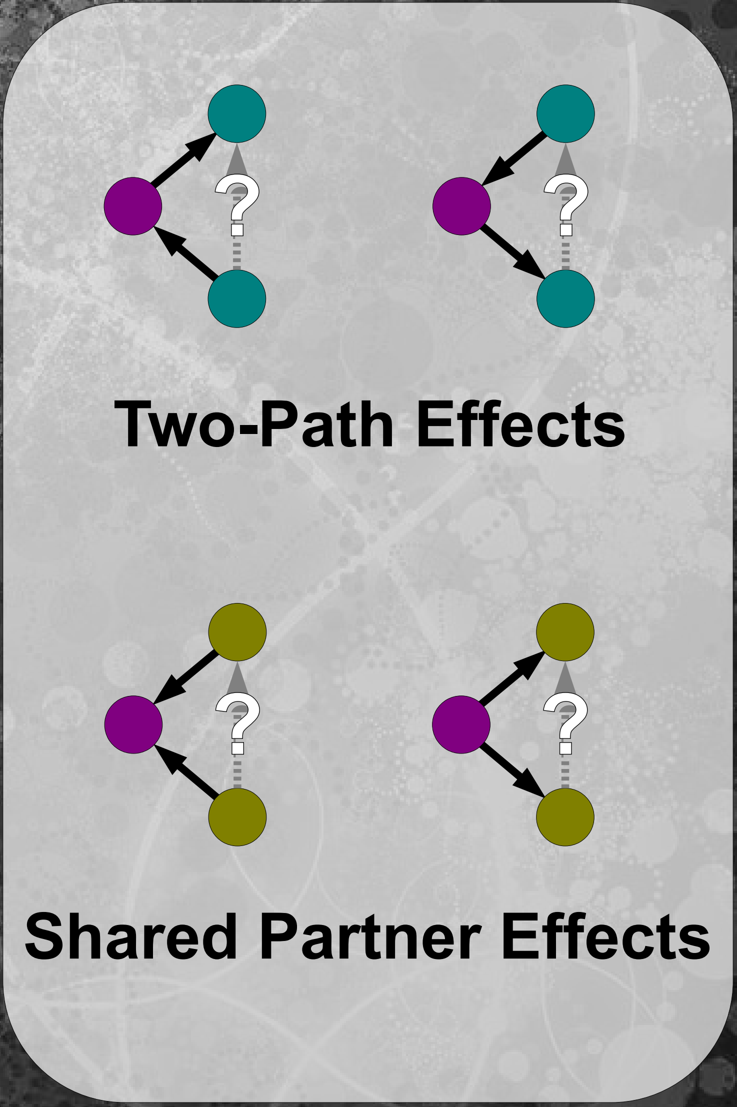
Recency/Ordering Effects

- Ordering of past contact potentially affects future contact
  - Reciprocity norms
  - Recency effects (salience)
- Simple parameterization: dyadic contact ordering effect
  - Previous incoming contacts ranked
    - Non-contacts treated as rank ∞
  - Log-rate of outgoing (i,j) contact adjusted by $\theta(1/\mathrm{rank}_{ji})$



Triadic/Clustering Effects

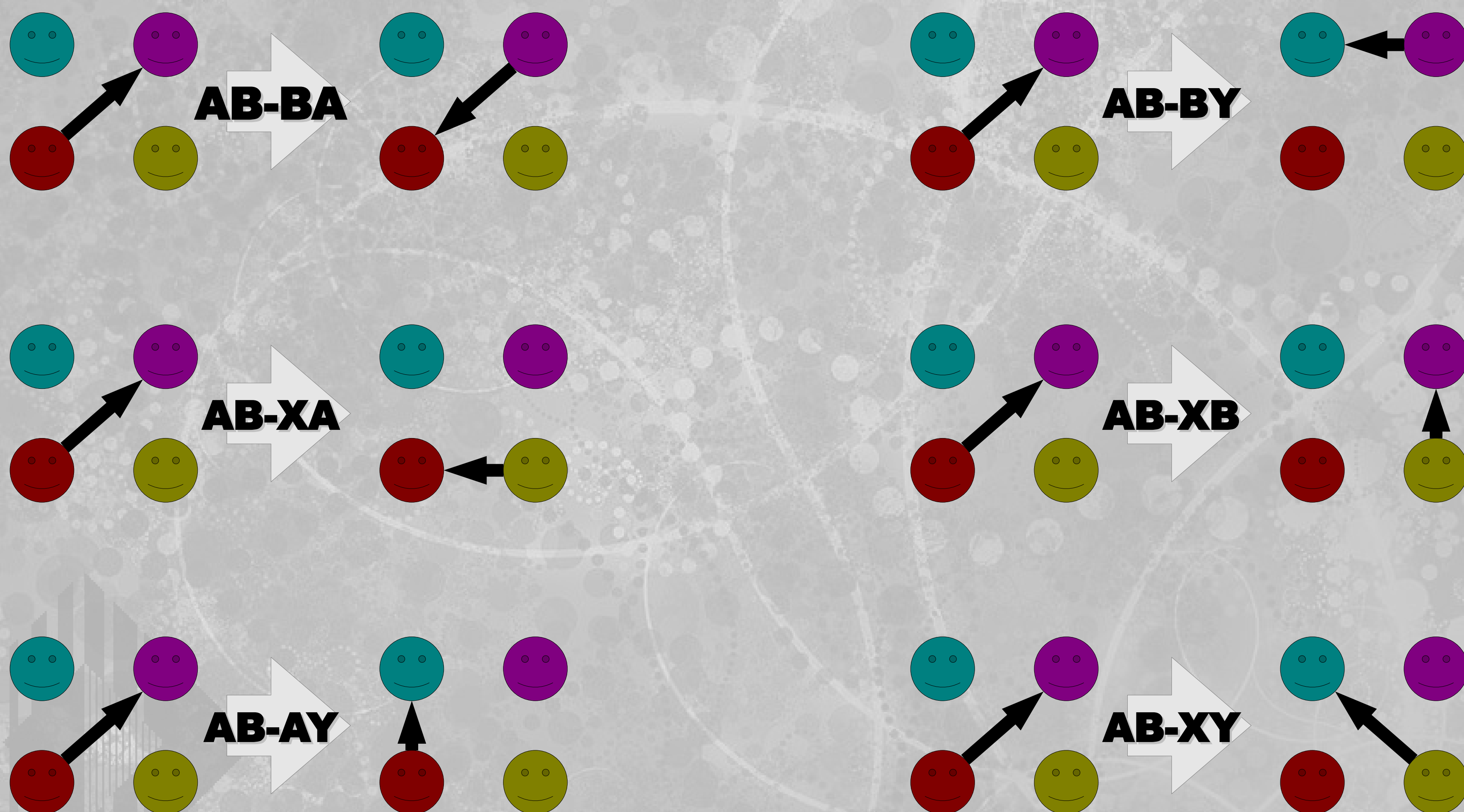
- Can also control for endogenous triadic mechanisms
  - Two-path effects
    - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
    - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)
  - Shared partner effects
    - Past outbound shared partners lead to/inhibit direct contact (common reference)
    - Past inbound shared partners lead to/inhibit direct contact (common contact)



Participation Shifts

- Proposal of Gibson (2003) for studying conversational dynamics
  - Classify actors into *senders*, *receivers*, and *bystanders*
  - When roles change, a *participation shift* ("P-shift") is said to occur
  - Study conversational dynamics by examining the incidence of P-shifts
- P-shift typology
  - For dyadic communication, 6 possible P-shifts; allowing indefinite targets expands set to 13
  - Can compute observed, potential shifts given an event sequence

Dyadic P-Shifts, Illustrated



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Preferential Attachment

- Past interactive activity affects tendency to receive action
  - E.g., emergent coordination roles
    - Exposure-based saliency ("who's out there?")
    - Practice/specialization (efficiency)
- Implement via past total degree effect on hazard of receipt
  - Fraction of all past calls due to *i* as effect for all *j* to *i* events