Modeling Relational Event Dynamics with statnet

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Overview

- **Content in a nutshell**
    - **Introduction to the use of relational event models (Butts, 2006; 2008) for the modeling interaction dynamics**
    - **Why this approach?**
        - **Fairly general**
        - **Principled basis for inference (estimation, model comparison, etc.) from actually existing data**
        - **Utilizes well-understood formalisms (event history analysis, multinomial logit)**
- **This workshop:**
    - **Introduction to modeling approach**
    - **Fitting dyadic relational event models**
    - **Model assessment and simulation**

Unpacking Networks: From Relationships to Action

- Conventional network paradigm: focus on temporally extensive relationships

- Powerful approach, but not always ideal

- Sometimes, we are interested in the social action that lies beneath the relationships....
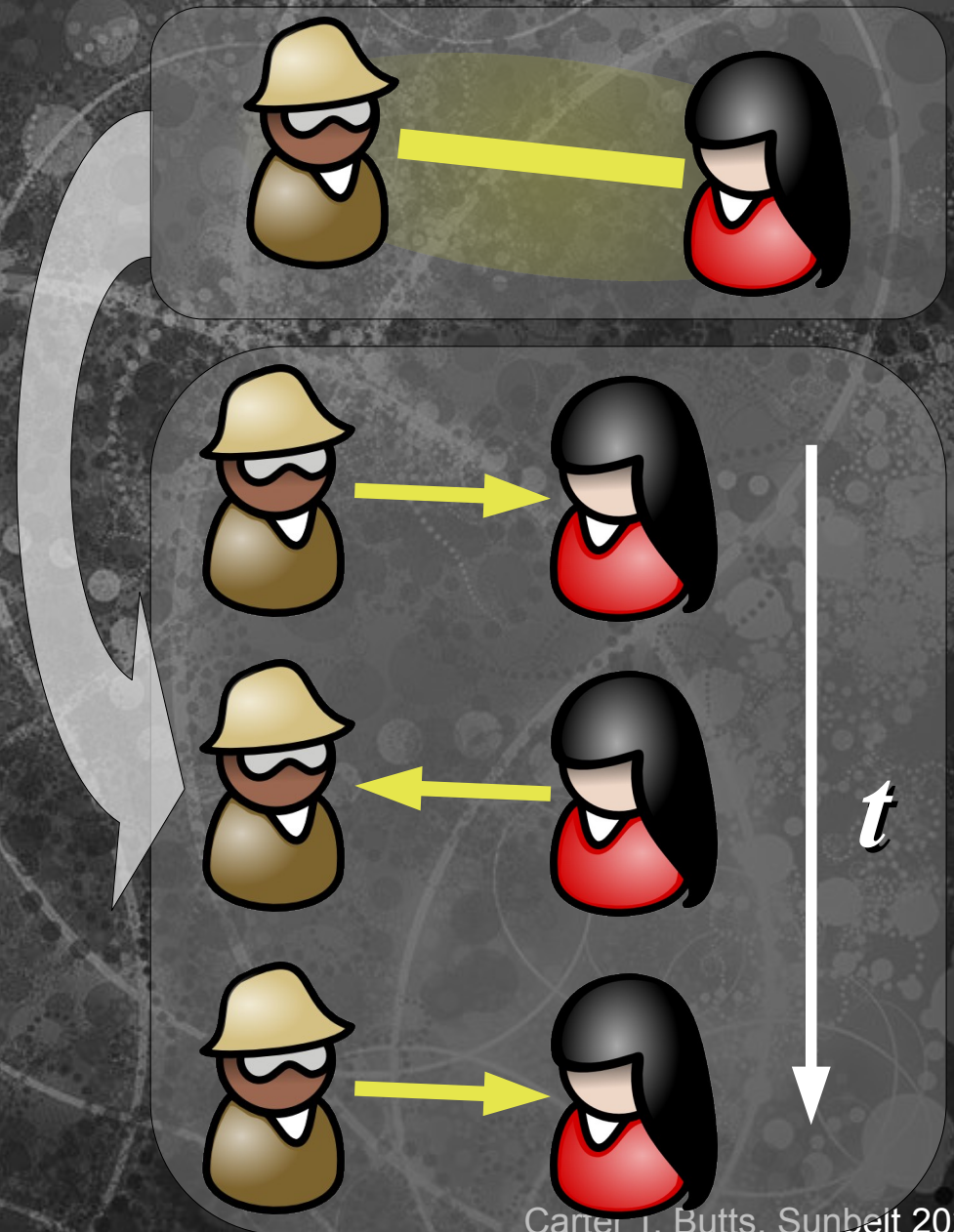
Actions and Relational Events

- **Action: discrete event in which one entity emits a behavior directed at one or more entities in its environment**
  - **Useful "atomic unit" of human activity**
  - **Represent formally by relational events**
- **Relational event:** $a=(i,j,k,t)$
  - $i \in \mathcal{S}$: **"Sender" of event** a; $s(a)=i$
  - $j \in \mathcal{R}$: **"Receiver" of event** a; $r(a)=j$
  - $k \in \mathcal{C}$: **"Action type" ("category") for event** a; $c(a)=k$
  - $t \in \mathbf{R}$: **"Time" of event** a; $\tau(a)=t$

Events in Context

- **Multiple actions form an event history,**
  $A_t = \{a_i : \tau(a_i) \leq t\}$

  - **Take a_0: $\tau(a_0)=0$ as "null action", $\tau(a_i) \geq 0$**

  - **Possible actions at t given by $\mathbf{A}(A_t) \subseteq \mathcal{S} \times \mathcal{R} \times \mathcal{C}$**

    - **Forms support for next action**

    - **Assume here that $\mathbf{A}(A_t)$ finite, constant between actions; may be fixed, but need not be**

- **Goal: model A_t**

  - **Treat actions as events in continuous time**

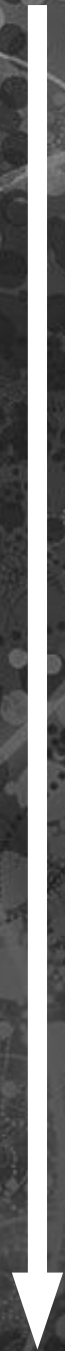 - **Hazards depend upon past history, covariates**
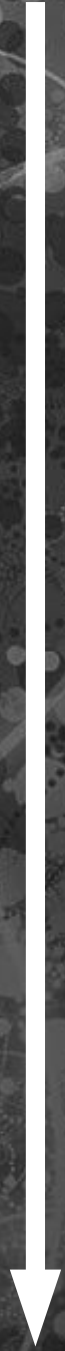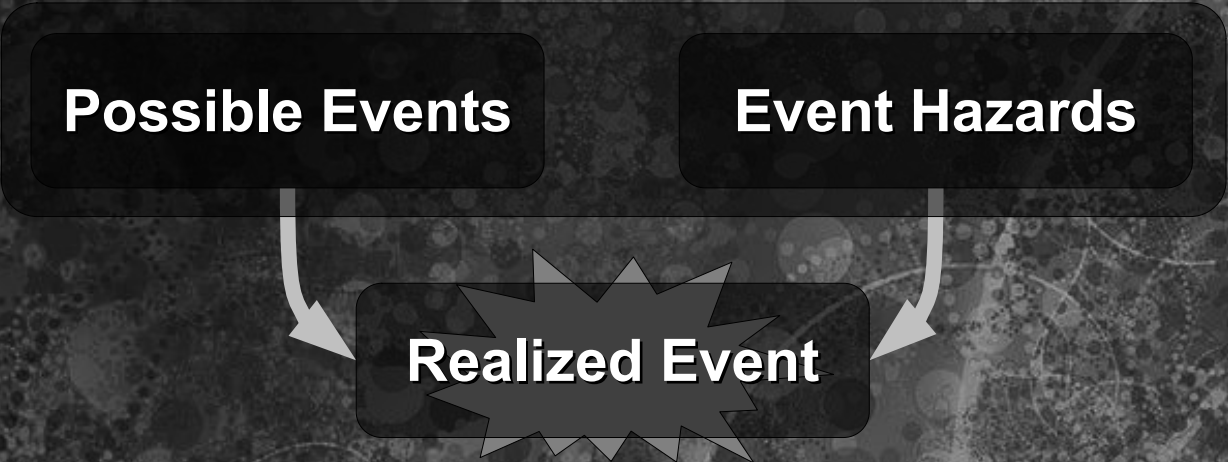
Possible Events   **Event Hazards**
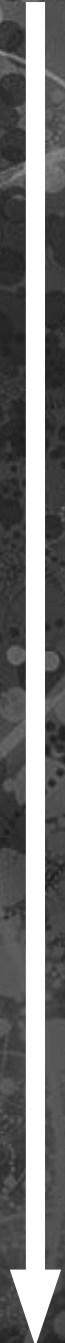
Time

Possible Events

Event Hazards

Context

Time

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Possible Events

Event Hazards

Realized Event

Time

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Possible Events **Event Hazards**

Realized Event

Possible Events **Event Hazards**
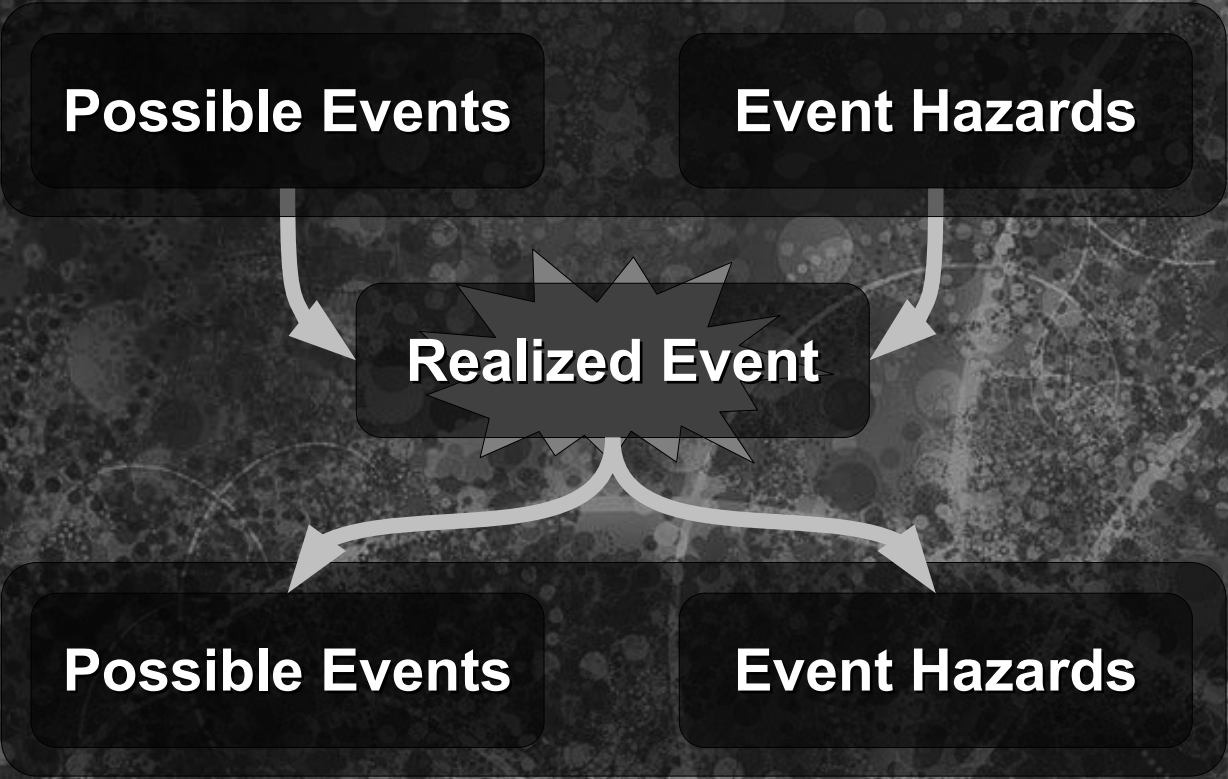
Realized Event

Time

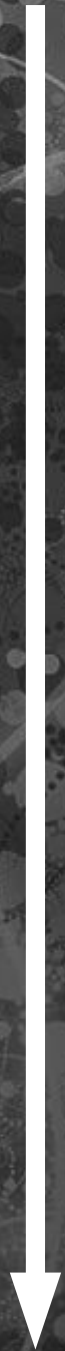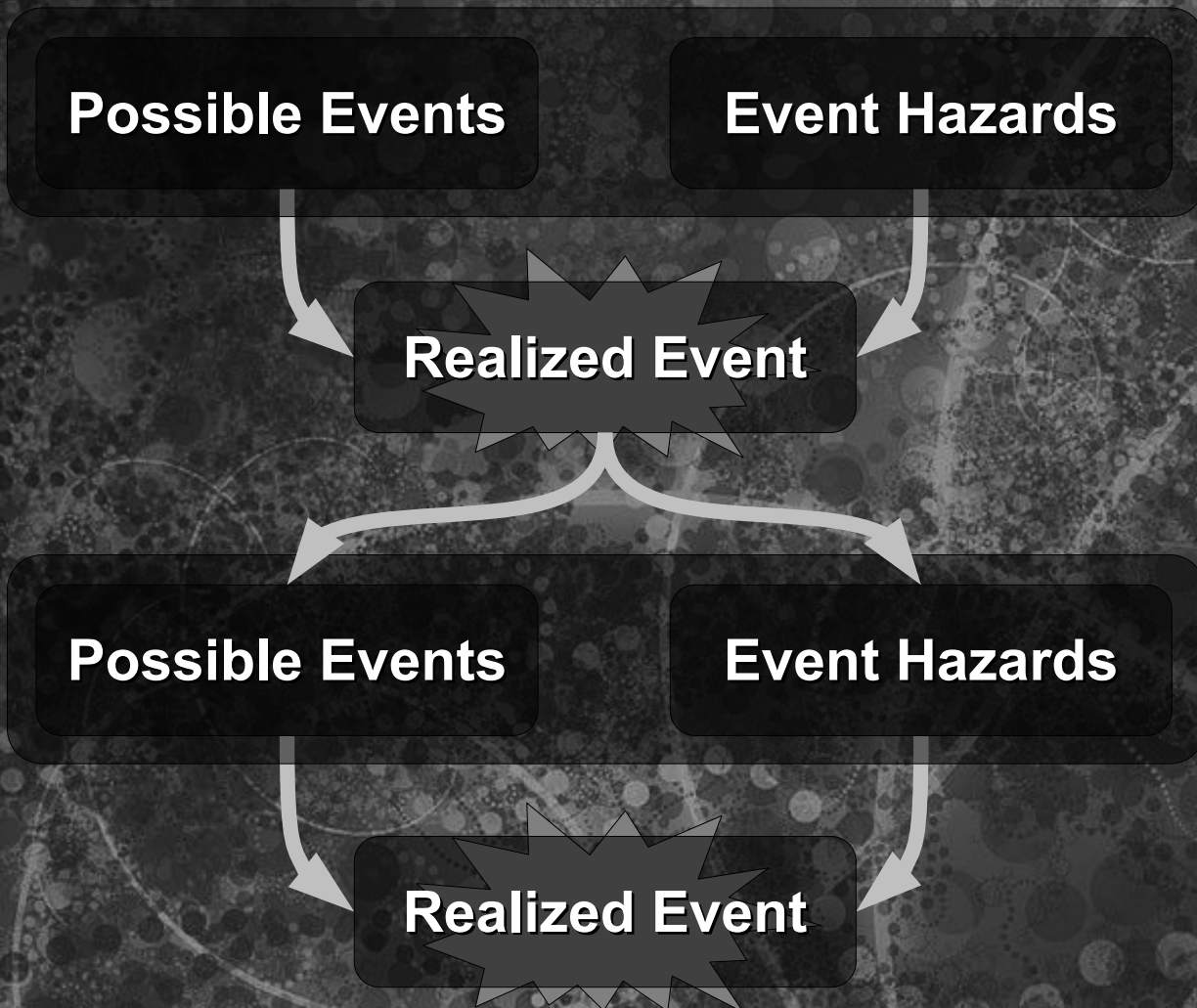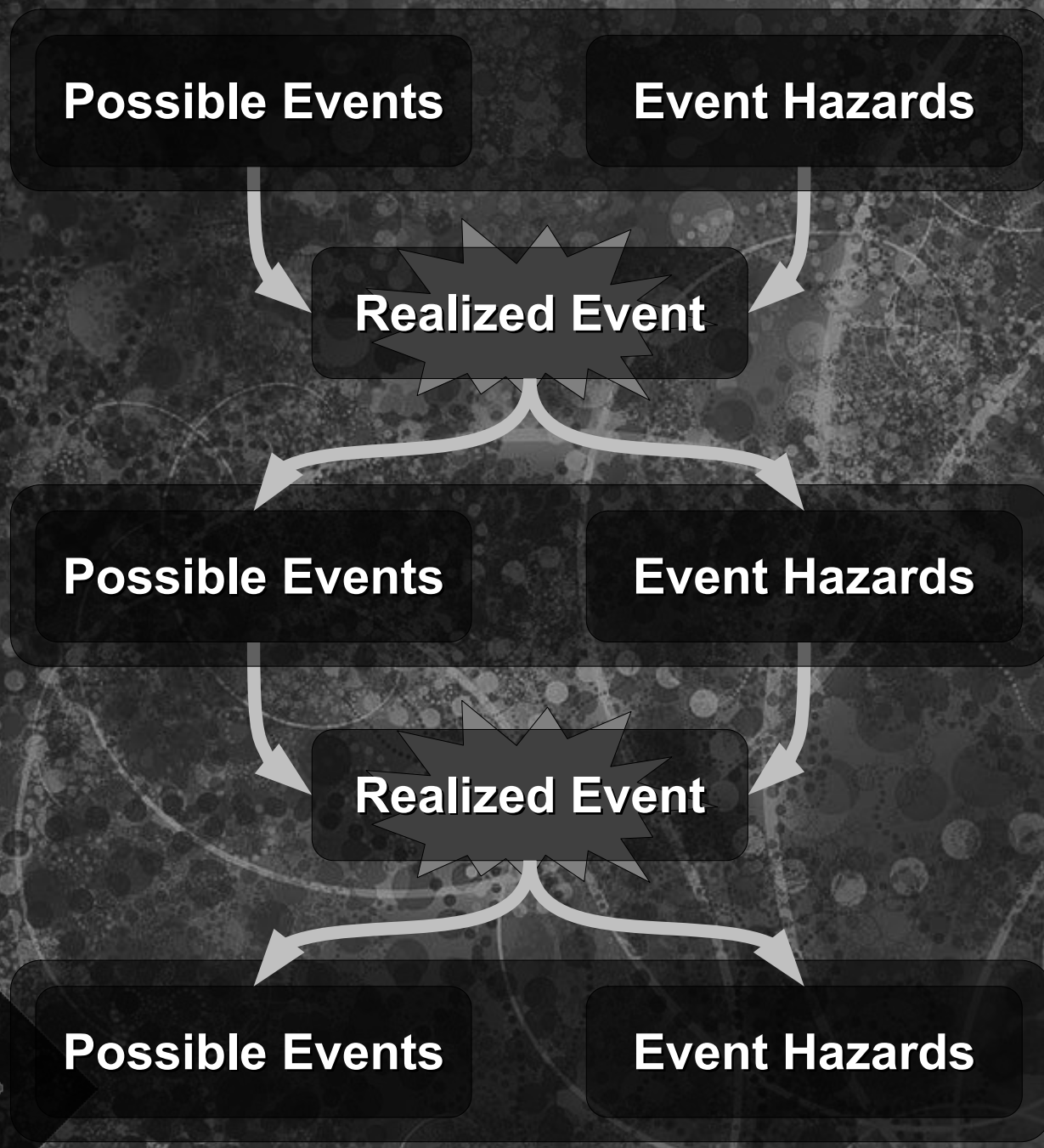Carter T. Butts, Sunbelt 2022

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Event Model Likelihood: Piecewise Exponential Case

- **Natural simplifying assumption: actions arise as Poisson-like events with piecewise constant rates**

  - Intuition: hazard of each possible event is *locally* constant, given complete event history up to that point

    - Waiting times conditionally exponentially distributed

    - Rates *can* change when events transpire, but not otherwise

      - Compare to related assumption in Cox prop. hazards model

    - Possible events likewise change only when something happens

- **Can use to derive event likelihood**

  - Let $M=|A_t|$, $\tau_i=\tau(a_i)$, w/hazard function $\lambda_{a_i A_k \theta}=\lambda(a_i, A_k, \theta)$; then

$$p(A_t|\theta) = \left[\prod_{i=1}^{M} \left(\lambda_{a_i A_{\tau_{i-1}} \theta} \prod_{a' \in A(A_{\tau_i})} \exp\left(-\lambda_{a' A_{\tau_{i-1}} \theta}\left[\tau_i - \tau_{i-1}\right]\right) \right) \right]\left[\prod_{a' \in A(A_t)} \exp\left(-\lambda_{a' A_t \theta}\left[t - \tau_M\right]\right) \right]$$

The Problem of Uncertain Event Timing

- **Likelihood of an event sequence depends on the detailed history**

    - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known

    - What if we only have (temporally) ordinal data?

- **Stochastic process theory to the rescue!**

    - Thm: Let $X_1,...,X_n$ be independent exponential r.v. w/rate parameters $\lambda_1,...,\lambda_n$. Then the probability that $x_i=\min\{x_1,...,x_n\}$ is $\lambda_i/(\lambda_1+...+\lambda_n)$.

    - Implication: likelihood of ordinal data is a product of multinomial likelihoods

        - Identifies rate function up to a constant factor

Event Model Likelihood: Ordinal Timing Case

- Using the above, we may write the likelihood of an event sequence A_t as follows:

$$p(A_t | \theta) = \prod_{i=1}^{M} \left[\frac{\lambda_{a_i A_{\tau_{i-1}} \theta}}{\sum_{a' \in \mathsf{A}(A_{\tau_i})} \lambda_{a_i A_{\tau_{i-1}} \theta}} \right]$$

- Dynamics governed by rate function, λ

$$\lambda_{a A_t \theta} = \begin{cases} \exp\left(\lambda_0 + \theta^T u\left(s(a), r(a), c(a), A_t, X_a\right)\right) & a \in \mathsf{A}(A_t) \\ 0 & a \notin \mathsf{A}(A_t) \end{cases}$$

- Where λ_0 is an arbitrary constant, $\theta \in \mathbb{R}^p$ is a parameter vector, and $u: (i,j,A_t,X) \rightarrow \mathbb{R}^p$ is a vector of statistics

Interpreting the Parameters

* In general, each unit change in u_i multiplies the hazard of an associated event by $\exp(\theta_i)$

    * For ordinal time case, unit difference in u_i adds unit of θ_i to log odds of a vs a'

* Connection to multinomial choice models

    * Let $\mathbf{A}_i(A_t)$ be the set of possible actions for sender i at time t. Then, conditional on no other event occurring before i acts, the probability that i's next action is a is given by

$$p(a|\theta) = \frac{\exp\left[\theta^T u\left(i, r(a), c(a), A_t, X_a\right)\right]}{\sum_{a' \in \mathbf{A}_i(A_t)} \exp\left[\theta^T u\left(i, r(a'), c(a'), A_t, X_{a'}\right)\right]}$$

Fitting Relational Event Models

- **Given A_t and u, how do we estimate θ?**

  - **Parameters interpretable as logged rate multipliers (in u)**

- **We have $p(A_t|\theta)$, so can conduct likelihood-based inference**

  - **Find MLE $\theta^* = \arg\max_\theta p(A_t|\theta)$, e.g., using a variant Newton-Rapheson or other method**

  - **Can also proceed in a Bayesian manner**

    - **Posit $p(\theta)$, work with $p(\theta|A_t) \propto p(A_t|\theta)p(\theta)$**

  - **Some computational challenges when |A| is large; tricks like MC quadrature needed to deal with sum of rates across support**

Persistence Effects

- **Inertia-like effect: past contacts may tend to become future contacts**
  - Unobserved relational heterogeneity
  - Availability to memory
  - (Compare w/autocorrelation terms in an AR process)
- **Simple implementation: fraction of previous contacts as predictor**
 - Log-rate of (i,j) contact adjusted by $\theta d_{ij}/d_i$
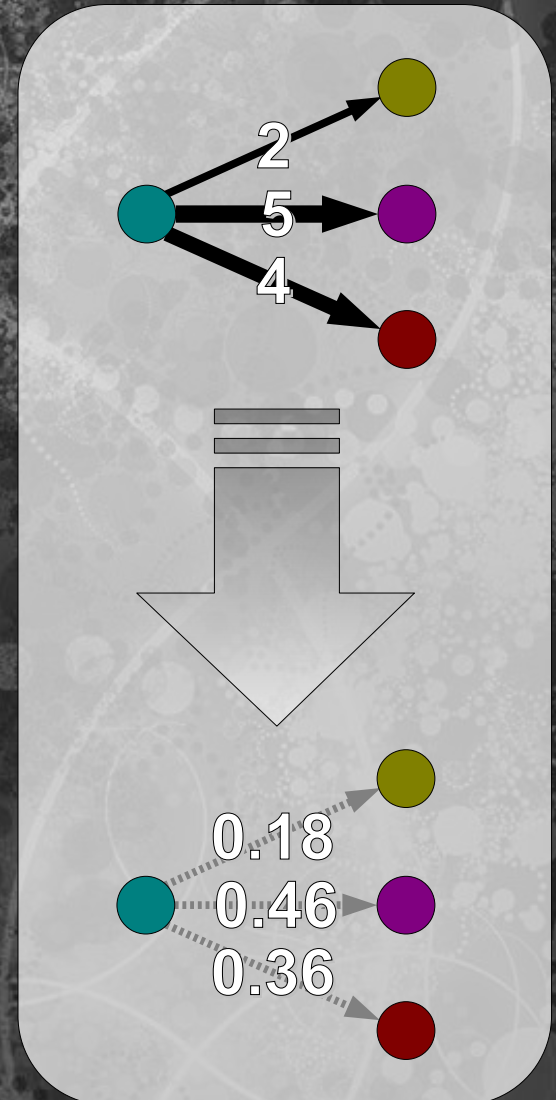
Recency/Ordering Effects

- **Ordering of past contact potentially affects future contact**
  - **Reciprocity norms**
  - **Recency effects (salience)**
- **Simple parameterization: dyadic contact ordering effect**
  - **Previous incoming contacts ranked**
    - **Non-contacts treated as rank ∞**
  - **Log-rate of outgoing (i,j) contact adjusted by $\theta(1/rank_{ji})$**
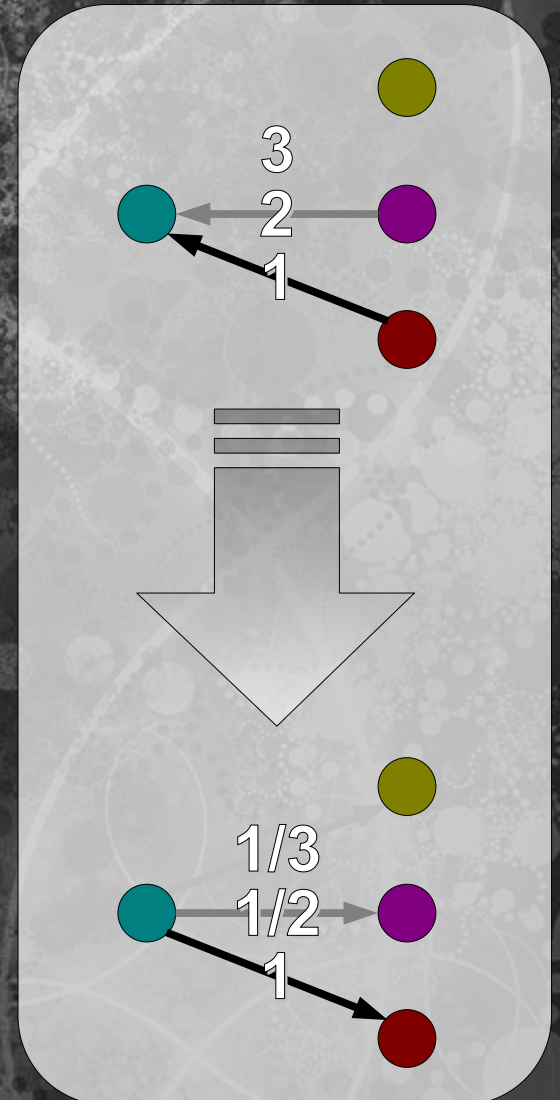
Triadic/Clustering Effects

- **Can also control for endogenous triadic mechanisms**
  - **Two-path effects**
    - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
    - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)
  - **Shared partner effects**
    - Past outbound shared partners lead to/inhibit direct contact (common reference)
    - Past inbound shared partners lead to/inhibit direct contact (common contact)
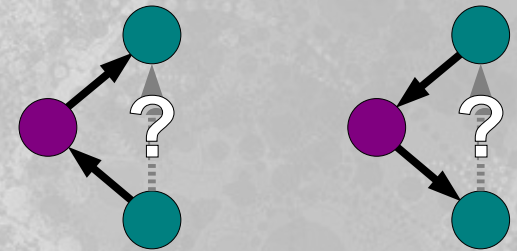

Two-Path Effects
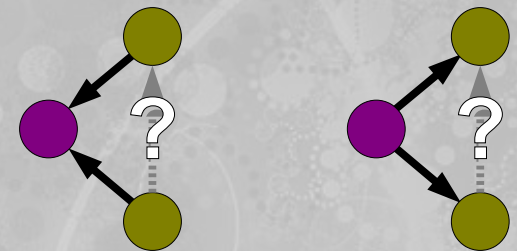
Shared Partner Effects

Participation Shifts

- **Proposal of Gibson (2003) for studying conversational dynamics**
    - Classify actors into *senders*, *receivers*, and *bystanders*
    - When roles change, a *participation shift* ("P-shift") is said to occur
    - Study conversational dynamics by examining the incidence of P-shifts
- **P-shift typology**
    - For dyadic communication, 6 possible P-shifts; allowing indefinite targets expands set to 13
    - Can compute observed, potential shifts given an event sequence

Dyadic P-Shifts, Illustrated
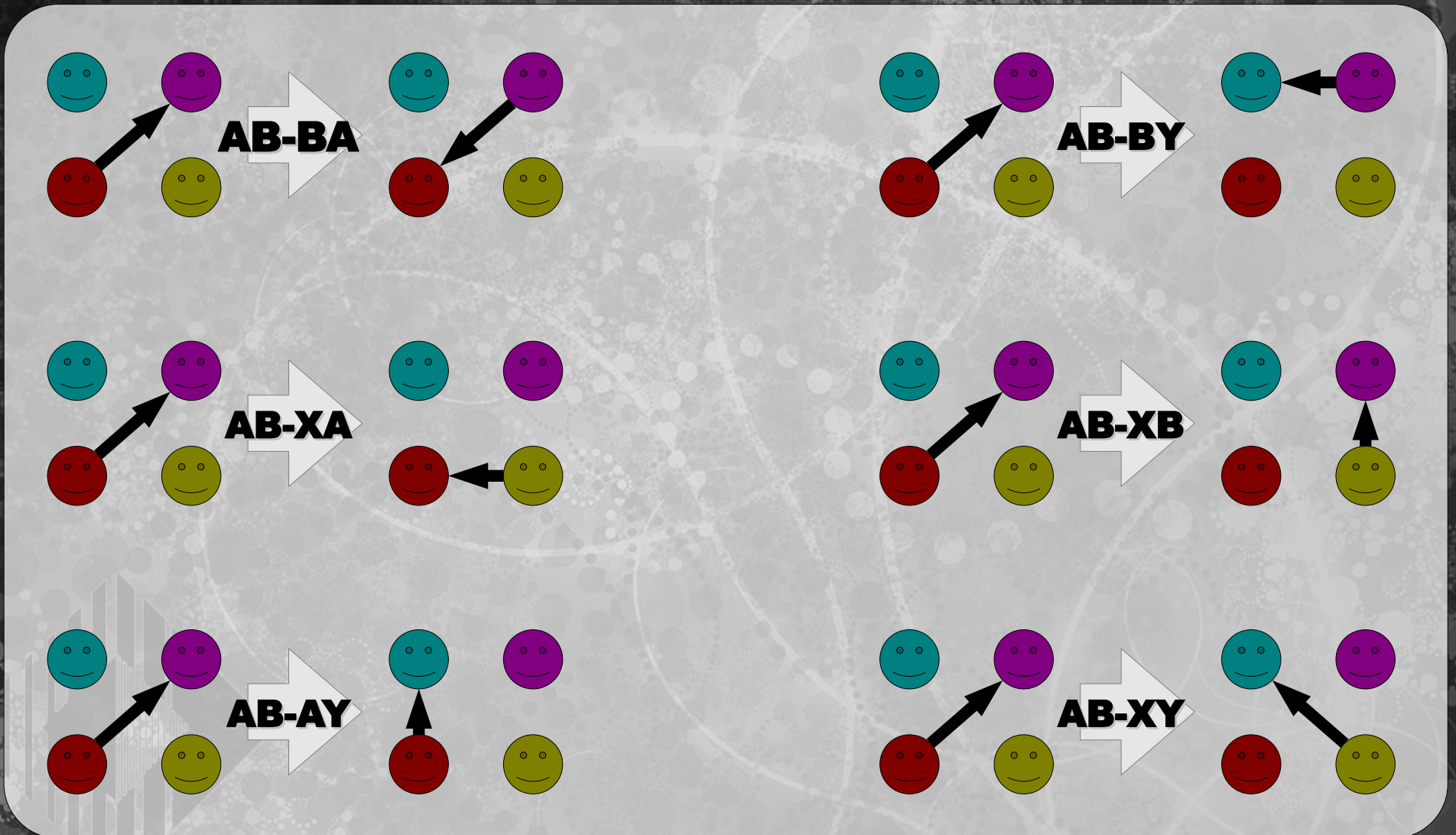
Preferential Attachment

- **Past interactive activity affects tendency to receive action**

  - **E.g., emergent coordination roles**

    - **Exposure-based saliency ("who's out there?")**

    - **Practice/specialization (efficiency)**

- **Implement via past total degree effect on hazard of receipt**

  - **Fraction of all past calls due to i as effect for all j to i events**