Modeling Relational Event Dynamics with statnet

Carter T. Butts
Department of Sociology and
Institute for Mathematical Behavioral Sciences
University of California, Irvine

Christopher S. Marcum
RAND Corporation

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Overview

- Content in a nutshell
  - Introduction to the use of relational event models (Butts, 2006; 2008) for the modeling interaction dynamics
  - Why this approach?
    - Fairly general
    - Principled basis for inference (estimation, model comparison, etc.) from actually existing data
    - Utilizes well-understood formalisms (event history analysis, multinomial logit)

- This workshop:
  - Introduction to modeling approach
  - Dyadic relational event models
  - Egocentric relational event models
  - Modeling complex event sequences
Unpacking Networks: From Relationships to Action

- Conventional network paradigm: focus on temporally extensive relationships
- Powerful approach, but not always ideal
- Sometimes, we are interested in the social action that lies beneath the relationships....
Actions and Relational Events

- **Action**: discrete event in which one entity emits a behavior directed at one or more entities in its environment
  - Useful "atomic unit" of human activity
  - Represent formally by relational events
- **Relational event**: \( a = (i, j, k, t) \)
  - \( i \in S \): "Sender" of event \( a \); \( s(a) = i \)
  - \( j \in R \): "Receiver" of event \( a \); \( r(a) = j \)
  - \( k \in C \): "Action type" ("category") for event \( a \); \( c(a) = k \)
  - \( t \in \mathbb{R} \): "Time" of event \( a \); \( \tau(a) = t \)
Multiple actions form an event history, 
\[ A_t = \{ a_i : \tau(a_i) \leq t \} \]

- Take \( a_0 : \tau(a_0) = 0 \) as "null action", \( \tau(a_i) \geq 0 \)
- Possible actions at \( t \) given by \( A(A_t) \subseteq S \times R \times C \)
  - Forms support for next action
  - Assume here that \( A(A_t) \) finite, constant between actions; may be fixed, but need not be

Goal: model \( A_t \)

- Treat actions as events in continuous time
- Hazards depend upon past history, covariates
Possible Events
Event Hazards

Context

Time

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Possible Events | Event Hazards
---|---
Realized Event

Time

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Event Model Likelihood: Piecewise Exponential Case

- Natural simplifying assumption: actions arise as conditionally independent, Poisson-like events with piecewise constant rates
  - Intuition: hazard of each possible event is \textit{locally} constant, given complete event history up to that point
  - Waiting times conditionally exponentially distributed
  - Rates \textit{can} change when events transpire, but not otherwise
    - Compare to related assumption in Cox prop. hazards model
    - Possible events likewise change only when something happens
- Can use to derive event likelihood
  - Let $M=|A|$, $\tau_i=\tau(a_i)$, w/hazard function $\lambda_{aiA_k}\theta = \lambda(a_iA_k, \theta)$; then
  
  \[ p(A_t|\theta) = \prod_{i=1}^{M} \left( \lambda_{a_iA_{\tau_i}} \prod_{a' \in A_{|A_{\tau_i}|}} \exp \left( -\lambda_{a'A_{\tau_i}} \theta \left( \tau_i - \tau_{i-1} \right) \right) \right) \prod_{a' \in A_{|A_t|}} \exp \left( -\lambda_{a'A_t} \theta \left( t - \tau_M \right) \right) \]
The Problem of Uncertain Event Timing

- Likelihood of an event sequence depends on the detailed history
  - Problem: exact timing is generally uncertain for many data sources (e.g., transcripts), though order is known
  - What if we only have (temporally) ordinal data?
- Stochastic process theory to the rescue!
  - Thm: Let $X_1, \ldots, X_n$ be independent exponential r.v. w/rate parameters $\lambda_1, \ldots, \lambda_n$. Then the probability that $x_i = \min\{x_1, \ldots, x_n\}$ is $\lambda_i / (\lambda_1 + \ldots + \lambda_n)$.
  - Implication: likelihood of ordinal data is a product of multinominal likelihoods
    - Identifies rate function up to a constant factor
Event Model Likelihood: Ordinal Timing Case

• Using the above, we may write the likelihood of an event sequence $A_t$ as follows:

$$p(A_t | \theta) = \prod_{i=1}^{M} \frac{\lambda_{a_i A_{t-i}} \theta}{\sum_{a' \in \mathbb{A}(A_{t-i})} \lambda_{a_i A_{t-i}} \theta}$$

• Dynamics governed by rate function, $\lambda$

$$\lambda_{a A_t \theta} = \begin{cases} 
\exp(\lambda_0 + \theta^T u(s(a), r(a), c(a), A_t, X_a)) & a \in \mathbb{A}(A_t) \\
0 & a \notin \mathbb{A}(A_t) 
\end{cases}$$

• Where $\lambda_0$ is an arbitrary constant, $\theta \in \mathbb{R}^p$ is a parameter vector, and $u: (i, j, A_t, X) \rightarrow \mathbb{R}^p$ is a vector of statistics.
Interpreting the Parameters

- In general, each unit change in $u_i$ multiplies the hazard of an associated event by $\exp(\theta_i)$
  - For ordinal time case, unit difference in $u_i$ adds unit of $\theta_i$ to log odds of $a$ vs $a'$

- Connection to multinomial choice models
  - Let $\mathbb{A}_i(A_t)$ be the set of possible actions for sender $i$ at time $t$. Then, conditional on no other event occurring before $i$ acts, the probability that $i$'s next action is $a$ is given by

$$p(a|\theta) = \frac{\exp[\theta^T u(i, r(a), c(a), A_t, X_a)]}{\sum_{a' \in \mathbb{A}_i(A_t)} \exp[\theta^T u(i, r(a'), c(a'), A_t, X_{a'})]}$$
Fitting Relational Event Models

- Given $A_t$ and $u$, how do we estimate $\theta$?
  - Parameters interpretable as logged rate multipliers (in $u$)
  - We have $p(A_t|\theta)$, so can conduct likelihood-based inference
    - Find MLE $\theta^* = \arg \max_{\theta} p(A_t|\theta)$, e.g., using a variant Newton-Raphesnon or other method
    - Can also proceed in a Bayesian manner
      - Posit $p(\theta)$, work with $p(\theta|A_t) \propto p(A_t|\theta)p(\theta)$
      - Some computational challenges when $|A|$ is large; tricks like MC quadrature needed to deal with sum of rates across support
Persisten3ce Effects

• Inertia-like effect: past contacts may tend to become future contacts
  • Unobserved relational heterogeneity
  • Availability to memory
  • (Compare w/autocorrelation terms in an AR process)

• Simple implementation: fraction of previous contacts as predictor
  • Log-rate of \((i,j)\) contact adjusted by \(\theta d_{ij}/d_i\)
Recency/Ordering Effects

- Ordering of past contact potentially affects future contact
  - Reciprocity norms
  - Recency effects (salience)
- Simple parameterization: dyadic contact ordering effect
  - Previous incoming contacts ranked
    - Non-contacts treated as rank $\infty$
  - Log-rate of outgoing $(i,j)$ contact adjusted by $\theta(1/\text{rank}_{ji})$
Can also control for endogenous triadic mechanisms

- Two-path effects
  - Past outbound two-path flows lead to/inhibit direct contact (transitivity)
  - Past inbound two-path flows lead to/inhibit direct contact (cyclicity)

- Shared partner effects
  - Past outbound shared partners lead to/inhibit direct contact (common reference)
  - Past inbound shared partners lead to/inhibit direct contact (common contact)
Participation Shifts

- Proposal of Gibson (2003) for studying conversational dynamics
  - Classify actors into senders, receivers, and bystanders
  - When roles change, a participation shift ("P-shift") is said to occur
  - Study conversational dynamics by examining the incidence of P-shifts

- P-shift typology
  - For dyadic communication, 6 possible P-shifts; allowing indefinite targets expands set to 13
  - Can compute observed, potential shifts given an event sequence
Dyadic P-Shifts, Illustrated

AB-BA

AB-XA

AB-AY

AB-BY

AB-XB

AB-XY
Past interactive activity affects tendency to receive action

- E.g., emergent coordination roles
  - Exposure-based saliency ("who's out there?")
  - Practice/specialization (efficiency)
- Implement via past total degree effect on hazard of receipt
  - Fraction of all past calls due to $i$ as effect for all $j$ to $i$ events