Exponential random graph modeling (ERGM)

Probability of observing a network (set of relationships) *y* on a given set of actors:

$$P(Y = y) = \frac{exp\{\theta' g(y)\}}{k(\theta)}$$

where: ***g(y)*** **= vector of network statistics (like the statistics in a standard regression)**

θ **= vector of model parameters (like the coefficients in a standard regression)**

$$k(\theta) = \sum_{\substack{all\ nets \\ on\ node \\ set\ of\ Y}} exp\{\theta' g(y)\}$$

Bahadur (1961), Besag (1974), Frank (1986); Wasserman and Pattison (1996)

Exponential random graph modeling (ERGM)

Given a network and a model (i.e. a set of g(y) statistics proposed to be of interest), one typically wants to maximum likelihood estimates of the *θ* coefficients for that model.

- The normalizing constant *k(θ)* makes this impossible to do directly.
- Main solution: Markov Chain Monte Carlo (Geyer and Thompson 1992, Crouch et al. 1998)

Then, given a model and its *θ* parameters, one typically wants to
(1) interpret those parameters
(2) assess the model fit
(3) generate network realizations with the specified probabilities

- The normalizing constant g(y) also makes (3) impossible to do directly.
- Main solution is also Markov Chain Monte Carlo.

Exponential random graph modeling (ERGM)

The statement about the probability of a network:

$$P(Y=y)=\frac{exp\{\theta' g(y)\}}{k(\theta)}$$

where: *g(y)* = vector of network statistics
θ = vector of model parameters
$$k(\theta)=\sum_{\substack{all\ nets\\ on\ node\\ set\ of\ Y}} exp\{\theta' g(y)\}$$

Is equivalent to the statement about the conditional probability of any tie in the network:

$$logit\ P\left(Y_{ij}=1 \middle| y_{ij}^{c}\right)=\theta'\left[z\left(y_{ij}^{+}\right)-z\left(y_{ij}^{-}\right)\right]$$

where: y_{ij} is the value of the tie from *i* to *j*
y_{ij}^{c} is the network *y*, excluding y_{ij}
y_{ij}^{+} is the network *y* with y_{ij} set to 1
y_{ij}^{-} is the network *y* with y_{ij} set to 0

Do not be fooled into thinking that this eliminates the need for estimation via MCMC by eliminating the normalizing constant.
But it does provide a useful framework for interpreting the parameters.