Exponential Random Graph Models for Social Networks

Introduction and Motivation

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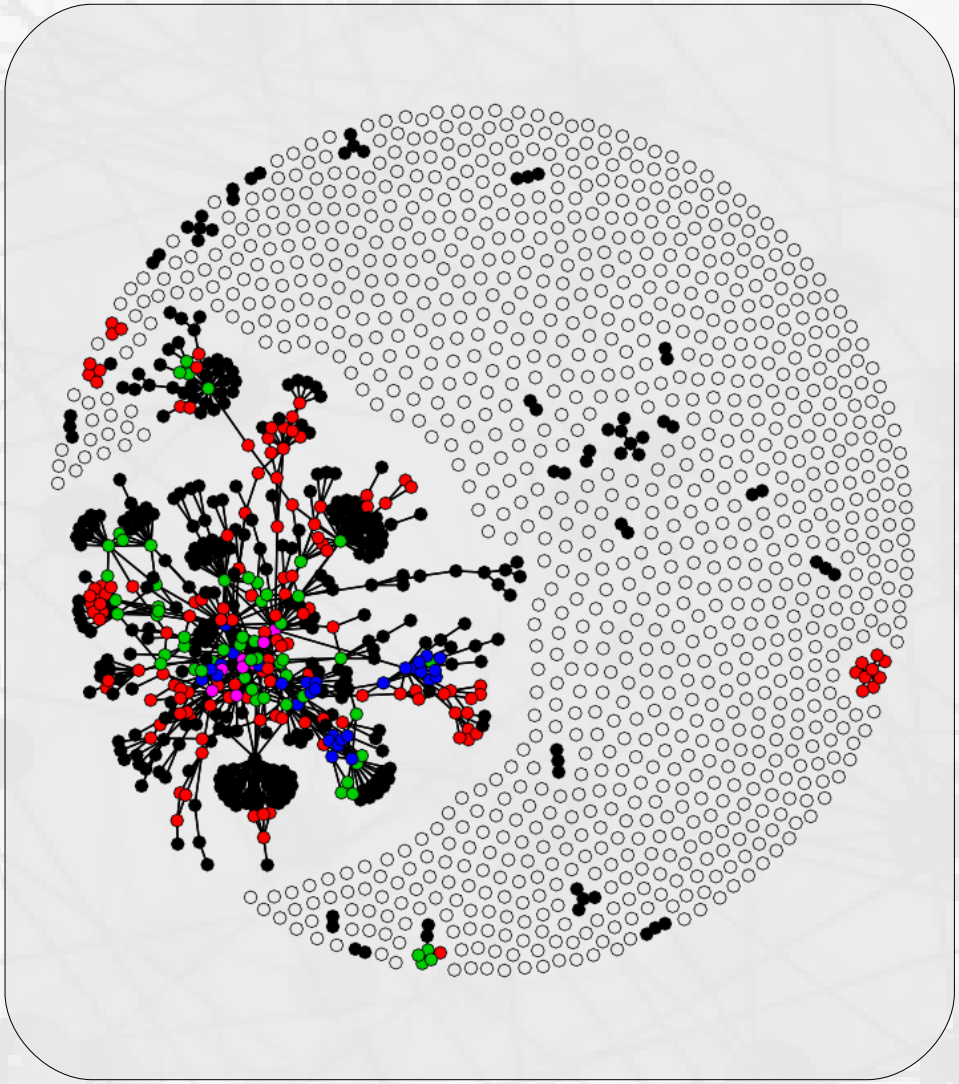
Welcome to the Workshop!

- **Main objectives**
  - Provide introduction to basic ERG methods for social network analysis
  - Emphasis on issues of practical data analysis
  - Touch on diagnostics,assessment
- **Tools**
  - Lecture
  - Handout
  - Software

Beginning at the Beginning

- **Many key questions regarding social systems are <u>relational</u>**
  - Connectivity, robustness, centrality, diffusion, influence, etc.
- **How do we answer such questions?**
- **The statistical approach:**
  - Assume that what we see reflects processes with many potential outcomes
  - Posit models that reflect our uncertainty about unknowns
  - Reason from observations and prior knowledge to unknown quantities in a principled manner

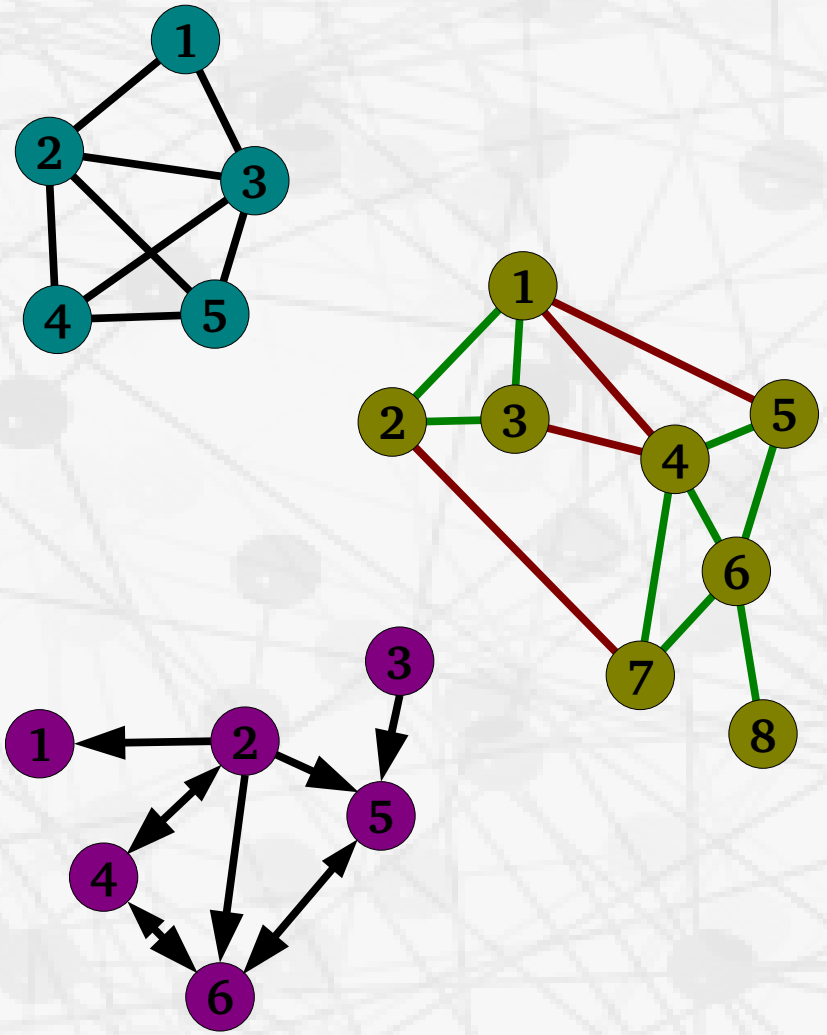


The Key Distinction: Focus on Relations

- **Relationship: an irreducible property of two or more entities**
  - Compare with properties of entities alone (“attributes”)
- **Focus: the properties and consequences of relations (rather than individual properties)**
  - Entities can be persons, non-human animals, groups, locations, organizations, regions, etc.
  - Relationships can be communication, acquaintanceship, sexual contact, proximity, migration rate, alliance/conflict, etc.
  - Social network analysis: the study of relational data arising from social systems
- **Leads to distinctive questions, challenges for data analysis (as we shall see!)**

Some Initial Vocabulary

- ***Network*: a collection of entities, together with a set of relations on those entities**
  - Entities: *nodes*, or *vertices*
  - Relations: *edges*, or *ties*
    - Focus on dyadic relations
    - Directed vs. undirected edges
    - May be signed or valued
- ***Graph*: a set of vertices together with a set of edges**
  - Mathematical representation of social structure

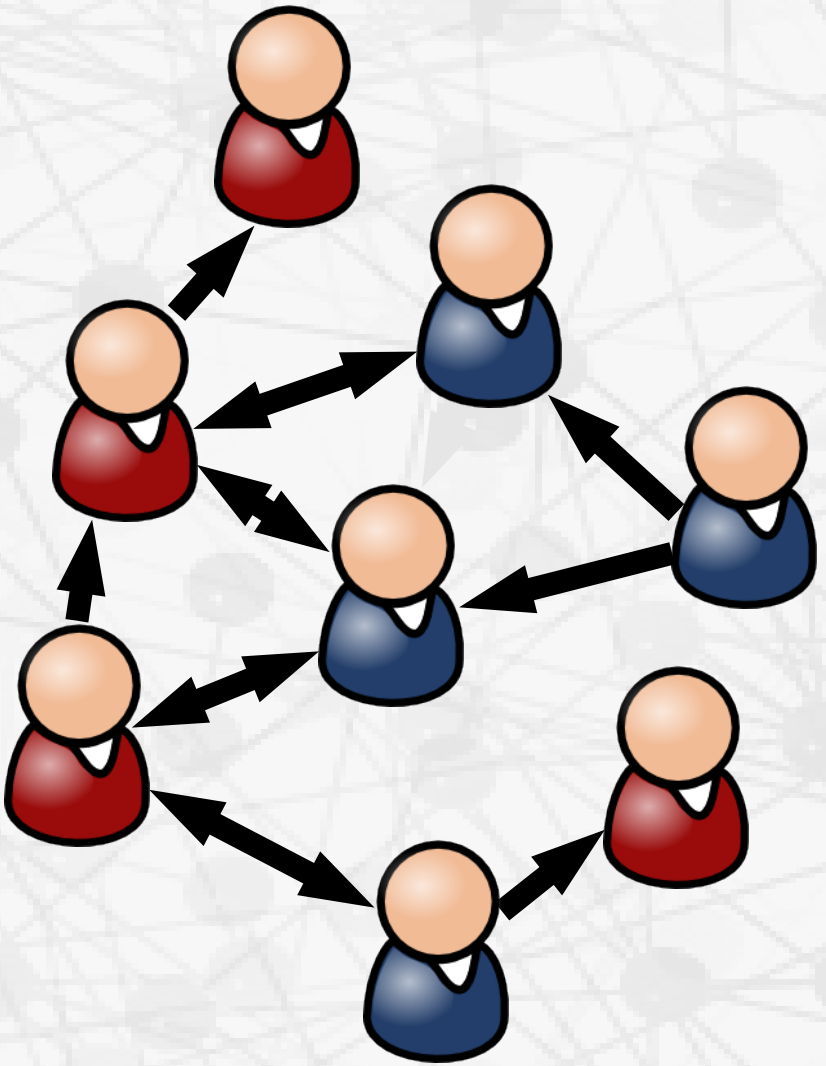


Why Statistical Models for Social (and Other) Networks?

- **Social systems are complex**
  - Many parts that affect each other
  - Substantial heterogeneity
- **Many mechanisms involved**
- **We're not good at measuring them**
  - Usually only see small chunks (and see above)
  - Error-prone observations
- **Upshot: the network we see may result from many mechanisms, plus noise and unobserved factors**
  - To draw conclusions about what is going on, must account for uncertainty
  - Predictions, conclusions should reflect this
  - Such goals require a statistical approach

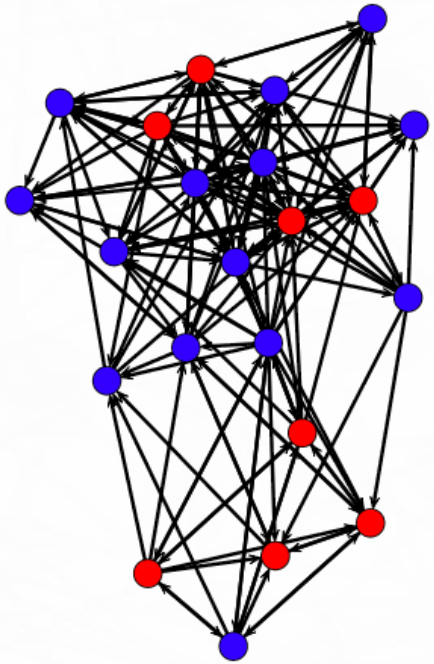
Motivating Example: The Reds and The Blues

- Consider a hypothetical community w/two groups - the "Reds" and the "Blues"
- Assume we are concerned with cooperation and trust in the community during a period of upheaval
- Our information is limited, but presume that we can observe networks of trust/friendship within representative subgroups....



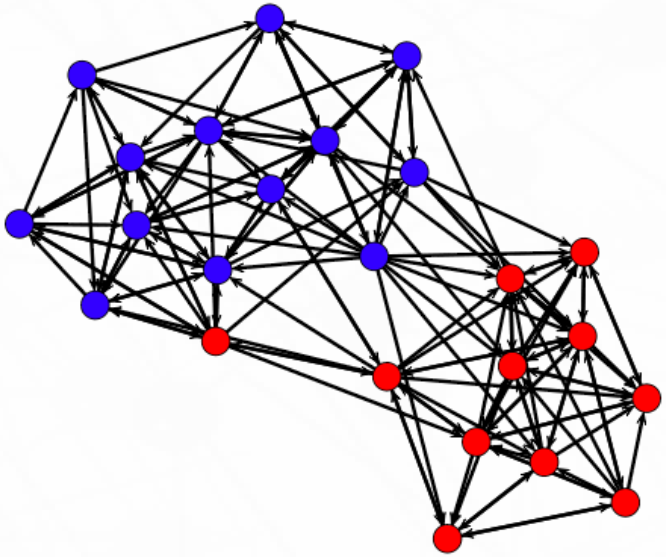
A Polarization Puzzle

Time 1



N=22

Time 2



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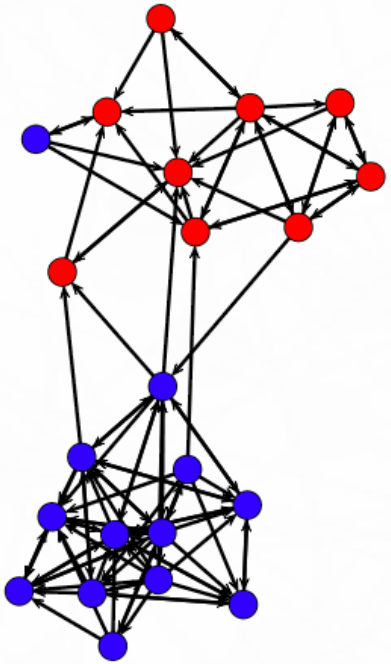
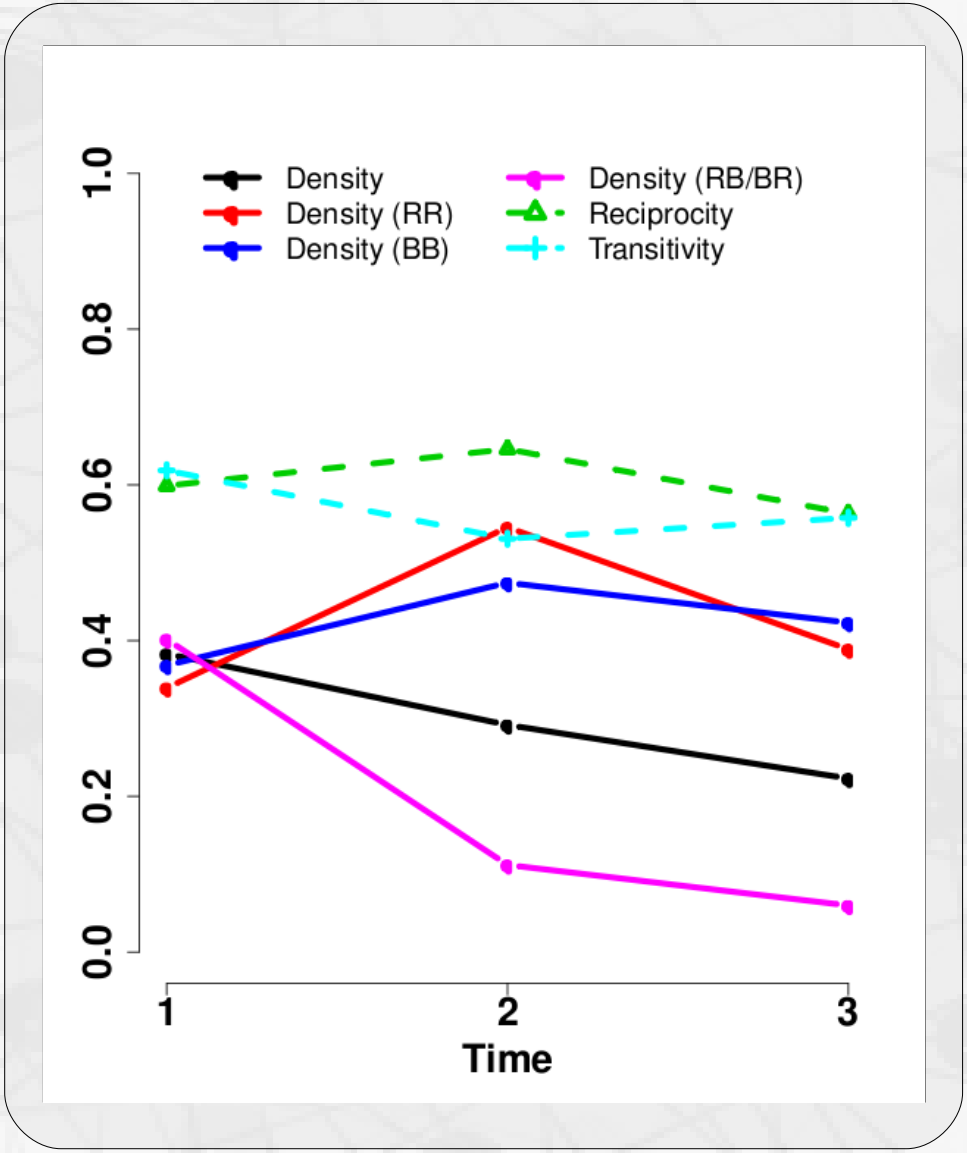
Time 3



N=22

First Step: Raw Descriptives

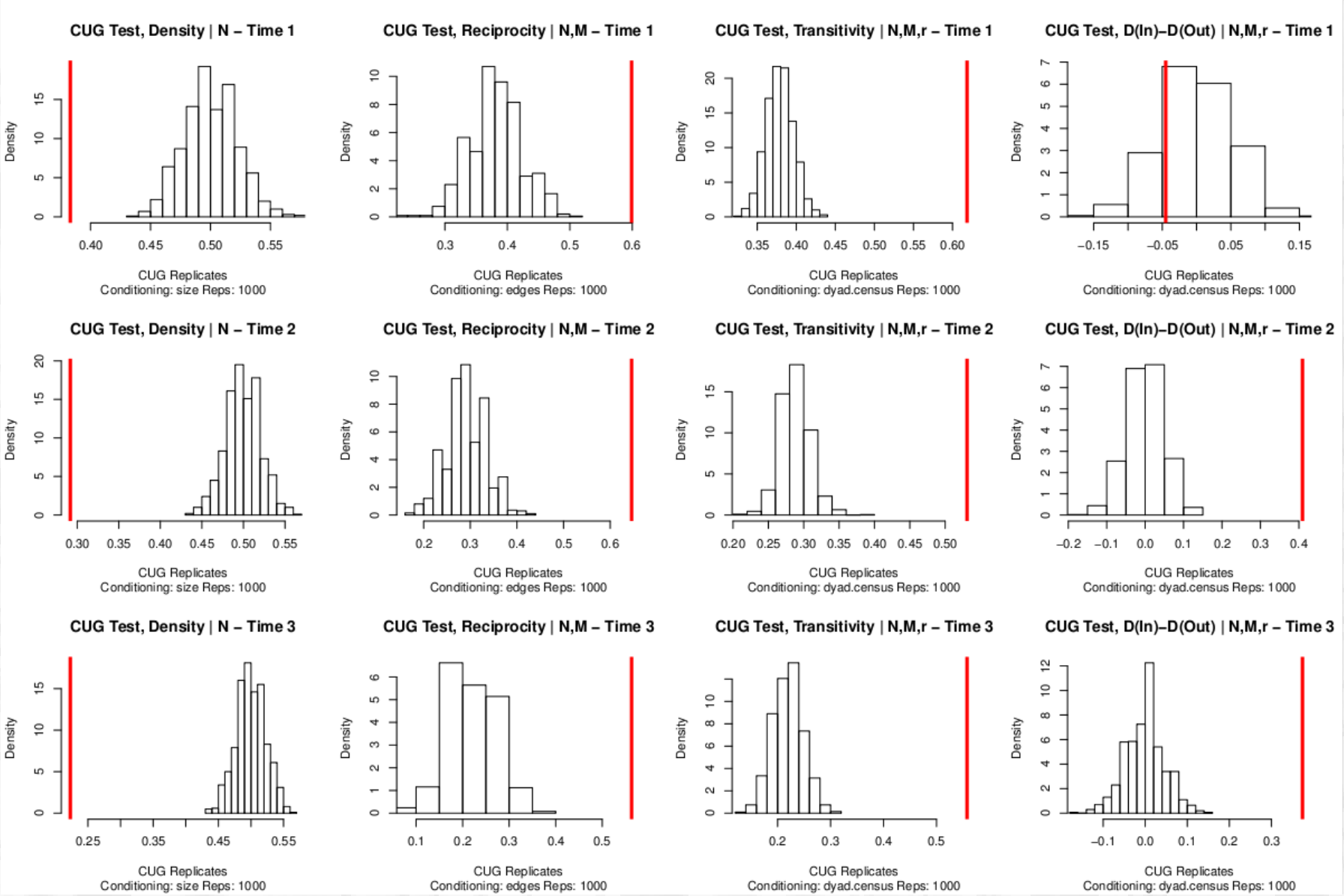
- **Without a statistical approach, one is limited to description**
- **Here, some typical examples:**
  - Density seems to fall slightly, although this masks an in/out-group difference
    - Red/Blue groups look similar
  - Moderately reciprocal, transitive networks, w/little change
- **Gives a more precise accounting of events, but not very insightful**
  - Are these changes even atypical of chance events?



Next Step: Baseline Models

- **Slight refinement: compare network properties to simple "baseline" models**
  - E.g., uniform random graphs, conditional on a few properties
- **Most elementary statistical approach**
  - Assesses whether combinatorics + elementary properties are sufficient to account for observations
- **Allows us to ask simple, marginal questions**
  - Is density atypical of population of all graphs given N?
  - Is reciprocity atypical of graphs given N,M?
  - Are transitivity, difference in in-group/out-group densities atypical of graphs given N,M,r?
- **Compare to classical null hypothesis testing**

Baseline Comparisons



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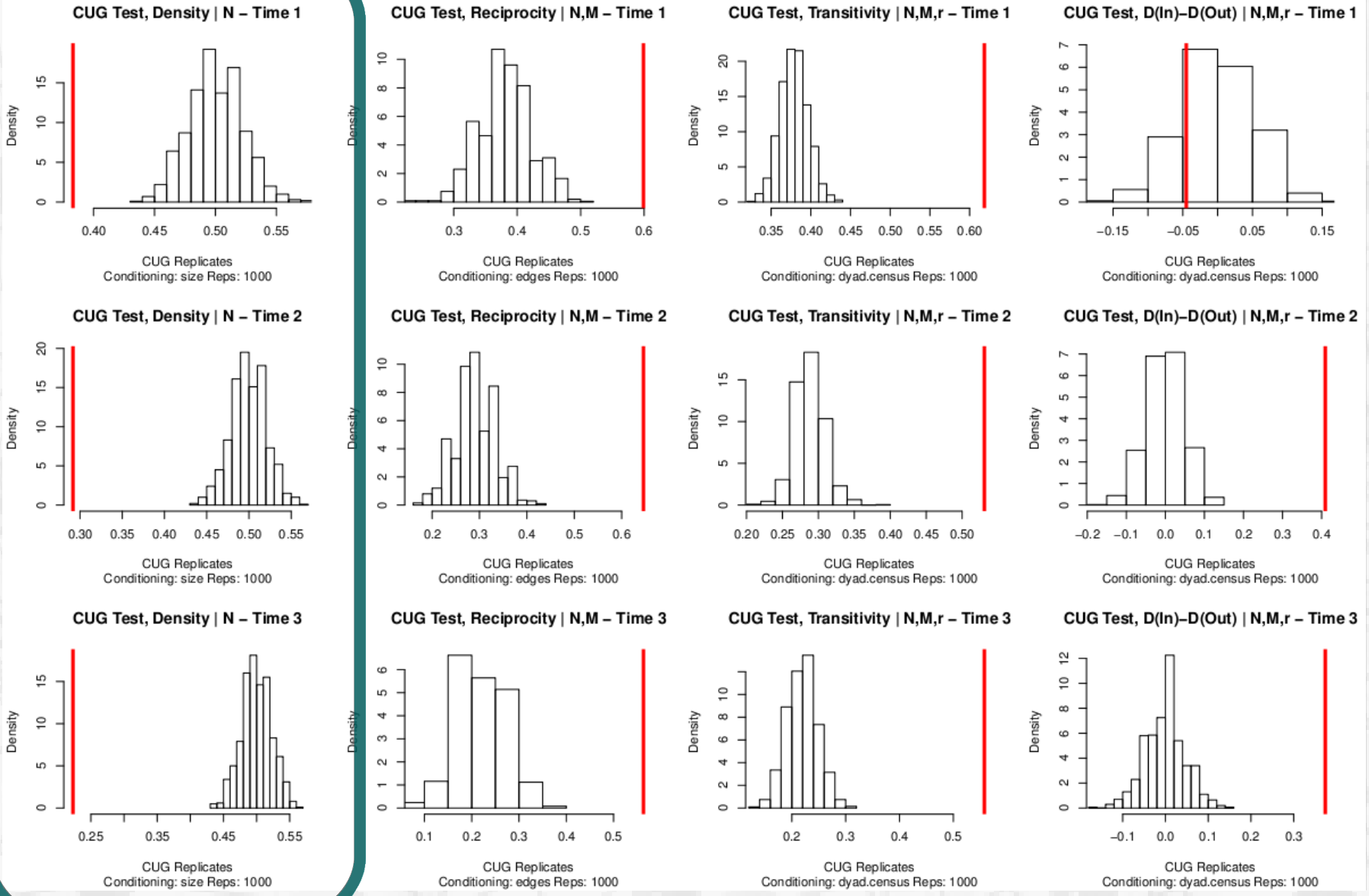

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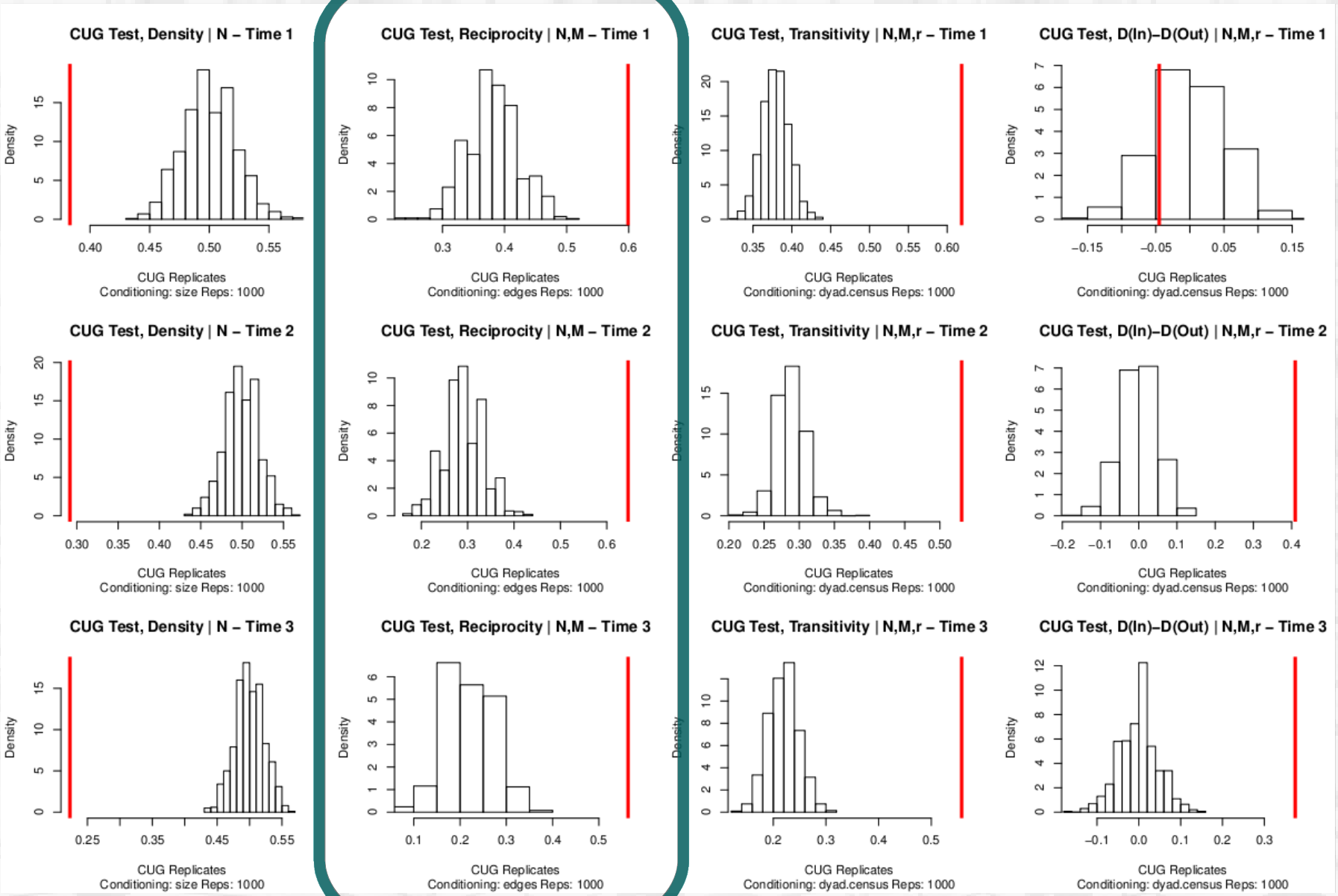

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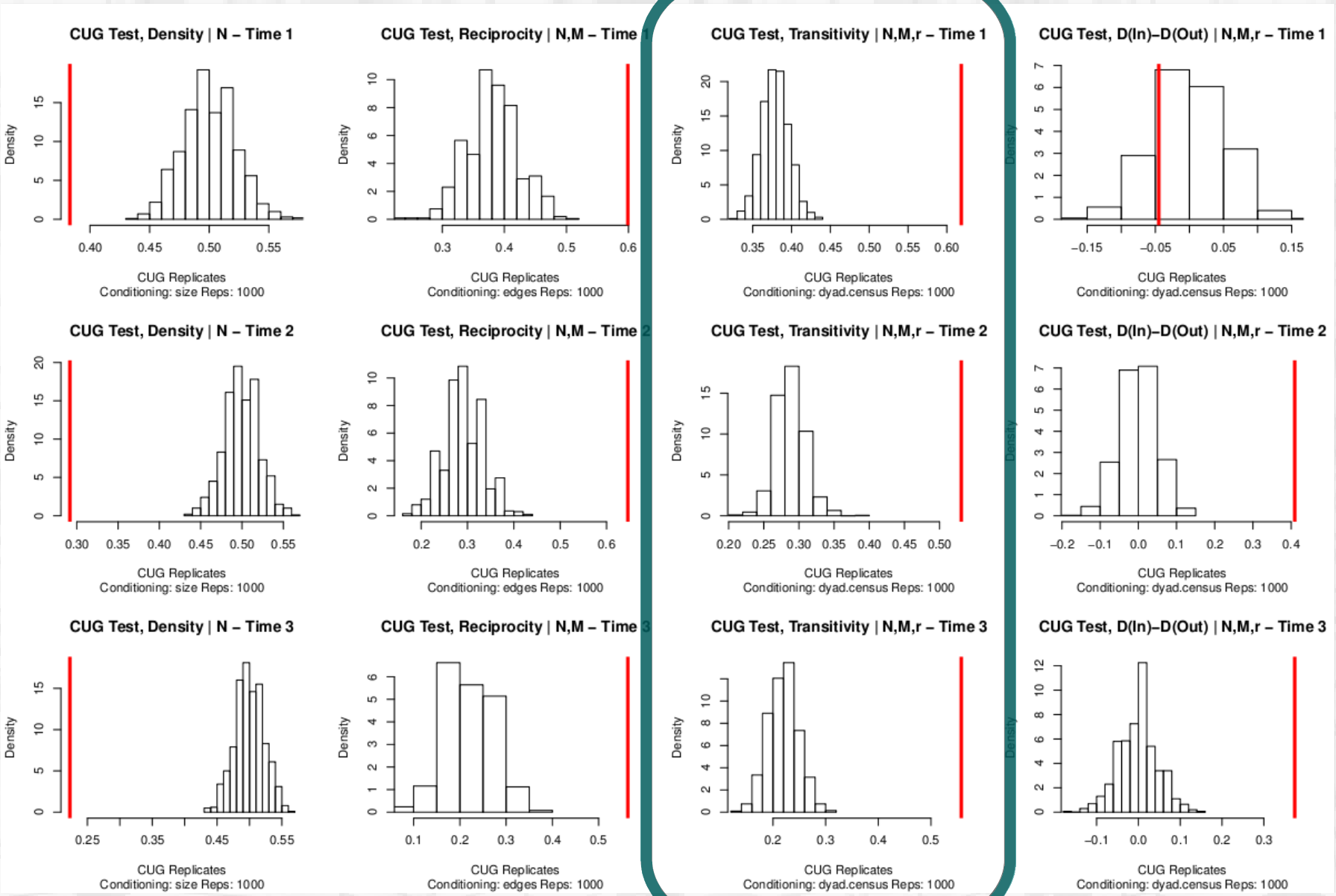

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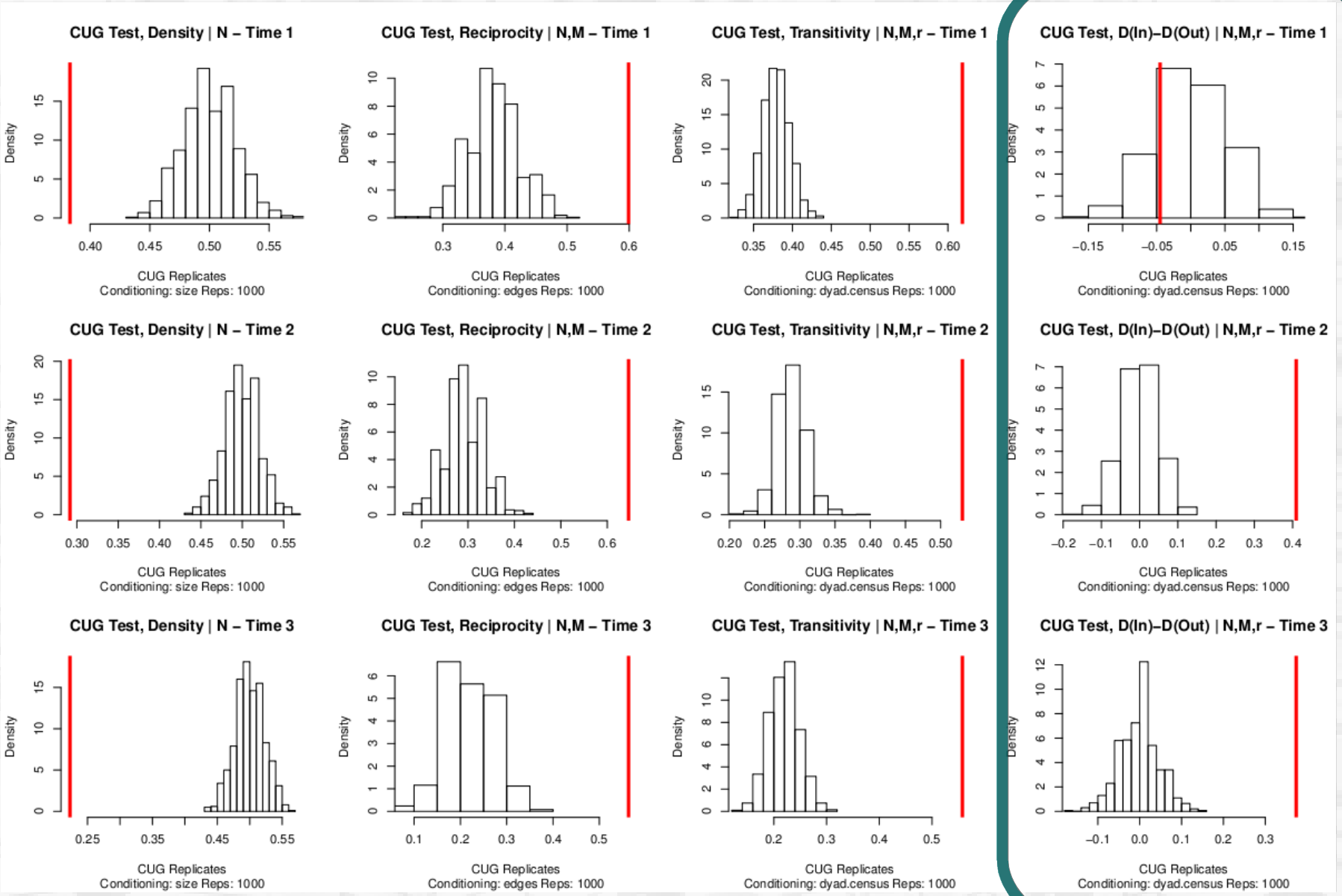
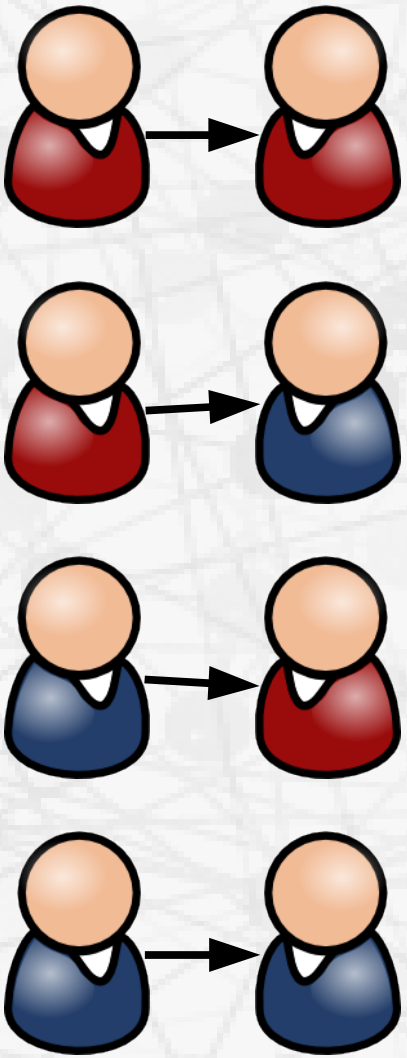


Beyond the Baselines

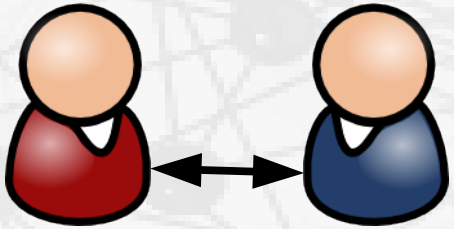
- **Baseline models only take us so far**
  - Few statistics lend themselves to conditioning
  - Difficult to look at multiple biases at once
  - Answers are qualitative in nature
  - Hard to account for sampling, error, etc.
  - Given "rejection" of the baseline, no clear path for further modeling
- **Solution: parametric models**
  - Identify candidate structural mechanisms
  - Parameterize using graph statistics
  - Fit models to data
    - Compare alternatives
    - Interpret parameter estimates
    - Assess adequacy
  - Can apply/extend for prediction, etc.

Sample Mechanisms

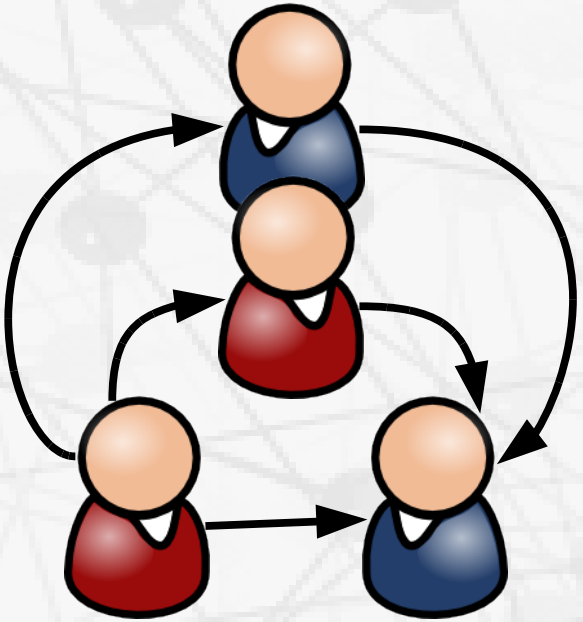
Heterogeneous Mixing



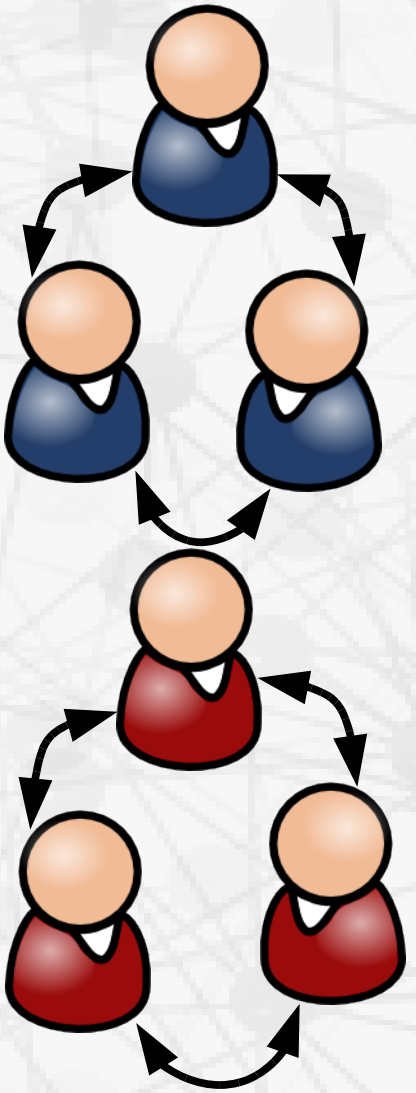
Mutuality Bias



Shared Partner Effects



Local Triangulation



Evaluating Competing Explanations

Edges	Mixing	Mutuals	GWESP	LocalTri	AIC	Rank
1	0	0	0	0	1777.684	15
1	1	0	0	0	1565.073	14
1	0	1	0	0	1516.578	13
1	0	0	1	0	1227.656	2
1	0	0	0	1	1478.532	12
1	1	1	0	0	1428.158	11
1	1	0	1	0	1279.456	6
1	1	0	0	1	1416.441	10
1	0	1	1	0	1234.932	3
1	0	1	0	1	1348.794	9
1	0	0	1	1	1290.241	7
1	1	1	1	0	1216.762	1
1	1	1	0	1	1339.640	8
1	1	0	1	1	1238.285	5
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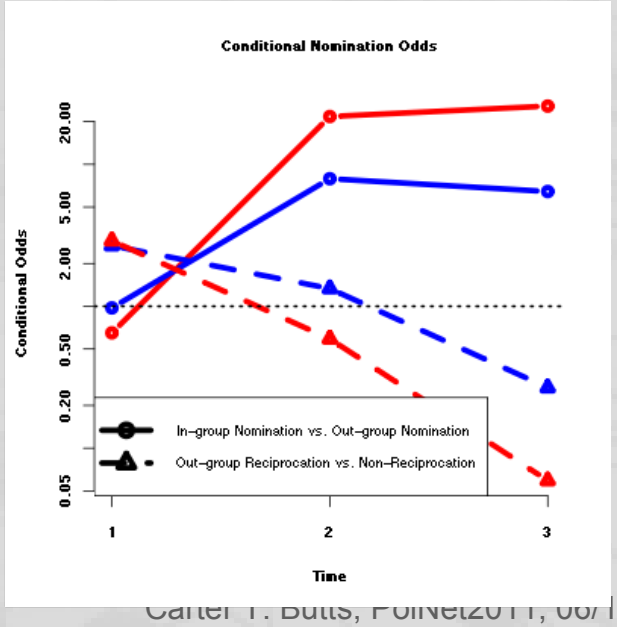
Interpreting the Mechanisms

	Time 1 MLE(SE)	Time 2 MLE(SE)	Time 3 MLE(SE)
Red→Red	-1.853 (0.291)	0.557 (0.226)	-1.069 (0.363)
Red→Blue	-1.421 (0.277)	-2.521 (0.428)	-4.317 (0.752)
Blue→Red	-1.501 (0.286)	-1.705 (0.354)	-2.809 (0.417)
Blue→Blue	-1.527 (0.198)	0.364 (0.226)	-0.948 (0.269)
Mutuals	2.484 (0.328)	1.992 (0.335)	1.489 (0.399)
GWESP	-0.030 (0.019)	-0.427 (0.031)	-0.018 (0.104)
GWESP(α)	1.218 (1.248)	0.744 (0.111)	0.598 (6.572)

- **Sharp decline in out-group nomination propensity w/out systematic in-group shift**
  - Conditional odds of in-group vs out-group nomination increase at time 2, stabilize
  - Effect somewhat stronger for Reds than Blues
- **Decline in mutuality**
  - Initially, both groups willing to conditionally reciprocate; by time 3, neither is!
- **No clear trend in third-party effects**
- **Overall: out-group prefs, reciprocity key**



Interpreting the Mechanisms

Time 1 MLE(SE) Time 2 MLE(SE) Time 3 MLE(SE)

(Incidentally, our "mystery" networks actually consist of attributions of friendship among 3rd, 4th, and 5th grade public school students, as collected by Parker and Asher (1993); "Reds" are females, and "Blues" are males. Any resemblance to the formation of competing factions within your place of business may or may not be completely accidental....)

- •Conditional odds of in-group vs out-group nomination increase at time 2, stabilize
  - •Effect somewhat stronger for Reds than Blues
- •Decline in mutuality
  - •Initially, both groups willing to conditionally reciprocate; by time 3, neither is!
- •No clear trend in third-party effects
- •Overall: out-group prefs, reciprocity key

And Beyond...

- **Given an initial model family, there is much more one can do**
  - Assess model adequacy versus target descriptives
  - Prediction (conditional, forecasting, scenario evaluation, etc.)
  - Extension/expansion given new data
- **These are difficult or impossible using a purely descriptive framework**
- **...Thus the promise of the statistical approach!**

Looking Ahead

- **An overview of today's topics:**
  - Quick introduction to/review of network data in R
  - Basic ERGM parametrization/estimation
  - Simulating draws from ERGMs
  - Model adequacy assessment
  - Diagnosis and repair of broken models
  - Additional functionality/open questions
- **Can find this and other material at polnet2011.statnet.org, www.statnet.org**