Model Degeneracy

- Using the faux.mesa.high network
- Fit a model where the formula is edges + triangle
  - What happens?

```
Error: Number of edges in a simulated network exceeds that in the observed by a factor of more than
20. This is a strong indicator of model degeneracy or a very poor starting parameter configuration.
If you are reasonably certain that neither of these is the case, increase the MCMLE.density.guard co
ntrol.ergm() parameter.
```

- Trying to fit this model, the algorithm heads off into networks that are *much* more dense than the observed network.

- What does this mean? That this model would not have produced this network, for any combination of parameter estimates for the two terms
  - i.e., this is a model misspecification problem

Degeneracy Plot (for the 2 star model)

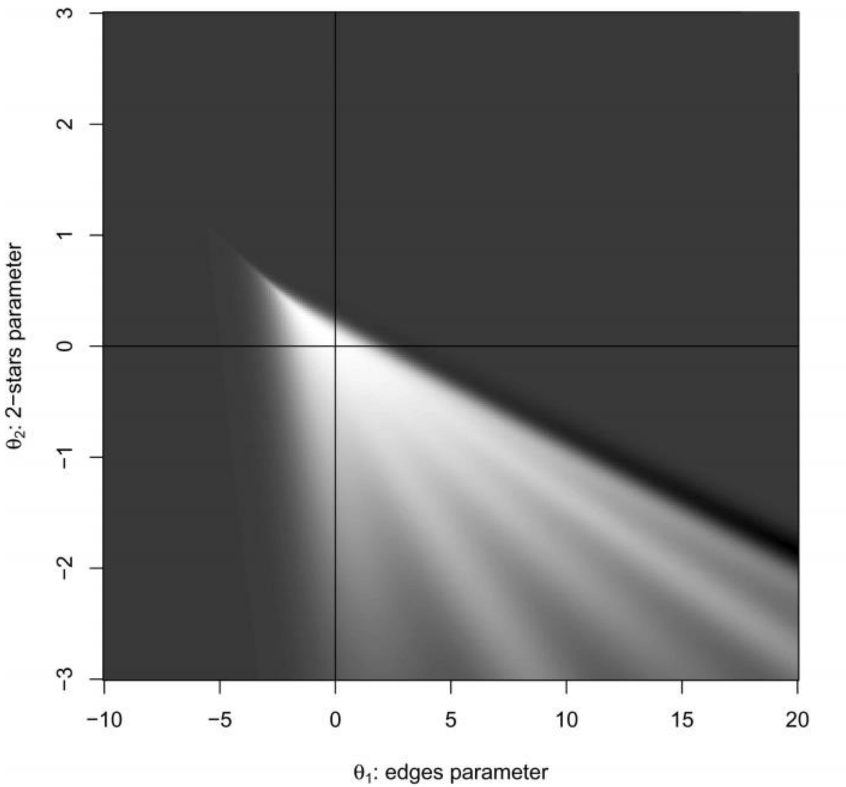


Figure 3: Cumulative Degeneracy Probabilities for graphs with 7 actors.

From Mark Handcock's 2003 tech report:
https://www.csss.washington.edu/Papers/2003/wp39.pdf

- Only the white area has networks with some interesting variation
- The dark areas are complete graphs, or empty graphs (+/- 1 or 2 edges)
- This model does not produce many useful networks

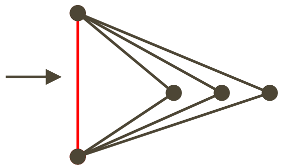
Solution: better network statistics

- Old statistic:
  - $t(x)$ = # of triangles in the graph

  Here, every additional 3-cycle has the same impact, θ

- New statistic:
  - Set declining marginal returns for each additional 3-cycle involving the same edge
  - I.e., the more partners in common that A and B have, the more likely they are to become partners—but the increase in the odds gets smaller with each additional shared partner
  - The specific function we place on this shared partner distribution involves a geometric weighting
  - Hence the name: ***geometrically weighted edge-wise shared partners***
  - A.k.a. GWESP
  - The parameter that specifies the rate of decline in marginal returns is α
  - The smaller the α, the more rapid the decline

Solution: better network statistics



$$gwesp = e^{\alpha}\sum_{i=1}^{n-2}\{1-(1-e^{-\alpha})^{i}\}sp_i$$

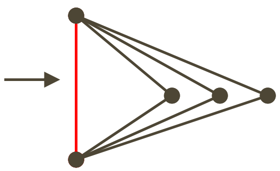
sp_i = # of edges with i shared partners

This configuration contains:

- 1 edge with 3 shared partners
- 6 edges with 1 shared partner

α	GWESP(α)				
0	$e^{0}[(1-(1-e^{-0})^{1})\times 6]$	+	$e^{0}[(1-(1-e^{-0})^{3})\times 1]$	=	7
0.5	$e^{0.5}[(1-(1-e^{-0.5})^{1})\times 6]$	+	$e^{0.5}[(1-(1-e^{-0.5})^{3})\times 1]$	=	7.55
1	$e^{1}[(1-(1-e^{-1})^{1})\times 6]$	+	$e^{1}[(1-(1-e^{-1})^{3})\times 1]$	=	8.03

Solution: better network statistics



$$gwesp = e^{\alpha}\sum_{i=1}^{n-2}\{1-(1-e^{-\alpha})^{i}\}sp_i$$

sp_i = # of edges with i shared partners

A count of each edge in each triangle (i.e. # of triangles *x* 3)

A count of edges in at least one triangle (because only an edge's first triangle counts)